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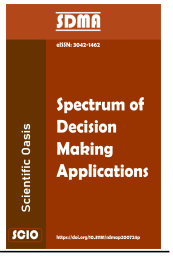

Constant Partition Dimension of Different Anticancer Drug Structures

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ABSTRACT

Cancer is a rapidly expanding number of unwelcome cells in the body. Carcinogens are substances that cause symptoms. A certain type of chemical molecule in cigarette smoke is known as a carcinogen. It can spread to several bodily parts. Some of this illness's indications and symptoms include a lump, unusual bleeding, a persistent cough, weight increase or decrease, etc. One of the biggest contributors to this malignant illness is tobacco chewing. Factors include obesity, a bad diet, laziness, and increasing alcohol consumption. Anticancer medications are used to cure this disease. In this work, we studied some of the anticancer medications in terms of partition dimension, where the partition resolving set is an improper subset to settle the entire atom set of a graph into a unique way to access each atom independently.

1. Introduction

The explosive growth of unwanted cells in the human body is cancer. Symptom substances are classified as carcinogens. A carcinogen is an organic compound found in cigarette smoke that contains certain components. It can spread to other places of the body. Some of the signs and symptoms of this illness include a lump, abnormal bleeding, a prolonged cough, and weight gain or loss, etc. Chewing tobacco is one of the main causes of this cancerous condition. Obesity, poor diet, laziness, and increased alcohol consumption are all factors. Several treatments for this deadly disease, like as surgery, radiation, and chemotherapy, can be used to treat it. Hormone therapy, targeted therapy, and other options are available. In these articles [**1**, **2**, **3**] anticancer medications are those that are used to treat the so-called cancer sickness.

In [**4**], authors discussed some medicinal structrues of COVID-19 antiviral disease in terms of graph

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theoretical parameters. This work motivates us to study some anti-cancer drugs given in [**5**], in terms of some graph theoretical parameters such as partition dimension. For this study, we need some assumptions to take care of before investigations or any type of calculations. In theoretical chemistry, drug structures are denoted by a molecular graph, an atom of a structure called a vertex, and an edge labelled from the links between atoms. Let $D(A, L)$ be a molecular graph of drug structure with atoms set A and links set L. All the chosen graphs or drug structures are simple i.e. without vertex to itself any edge and without two edges in any two vertices. For further assumptions, basic definitions, and theoretical study of molecular graph we refer to see [**6**, **7**, **8**, **9**].

2. Literature Review

In the research work of [**10**], the Authors discussed windmill graphs in terms of metric basis and their generalization. [**11**], Authors discussed the generalized version of the metric dimension graph and they defined this parameter on two variables. [**12**], Researchers computed metrics and their upper bounds on some generalized families of graphs. In this [**13**], Polycyclic hydrocarbons are detailed with the concept of metric and their generalizations. In this [**14**], symmetric graphs are obtained by the Rooted product and they studied metrics and their generalizations. [**15**], Researchers give the idea of Hollow Coronoid on metric dimension and their Generalization.[**16**], Measured the Resolvability of Quartz structure, and particularly they computed metric parameters for the structure of quartz without considering the pendant nodes outside the circle. In this [**17**], detail the rough graphs on the topics of metric dimensions and their generalized parameters. [**18**], Authors discussed hereditary bipartite and computing the metric basis of this generalized class of complex networks. In this [**19**], The idea of pseudo valuation on KU-algebras and investigated, The relationship between pseudo-valuations and KU-algebras and their generalizations. For more recent articles on chemical networks and metric parameters of different chemical structures and networks are available in [**20**, **21**, **22**, **23**, **24**, **25**, **26**, **27**, **28**].

Here are some citations about the partition dimensions and some are related to the metric dimension calculated in [**18**] and prove that in a cycle graph, there is no change in metric dimension by adding an edge in the cycle graph. The partition dimension of the fullerene graph is 3 calculated in [**29**]. The constant partition dimension of the hexagonal and honeycomb network was calculated in [**30**]. In [**31**] some wheel-related graphs are computed of partial dimension. Some works on trees and uni cyclic graphs and derived the bonds on partial dimension according to [**32**, **33**]. In [**34**] described the partition dimension $n-3$ of graphs. In [**35**] Some graphs are calculated by the sum operation of cycle and path graphs to obtain the partition dimension. In [**36**] and [**37**] calculate the partition dimension of circulant graph. In [**38**] calculated it complete multipartite graphs.

3. Methodology: Preliminaries and Background

In this section, we will discuss the methodology of our chosen topic in the form of pure mathematical definitions. Very first, the locating set was laid down its foundation by Slater in 1975 [**39**], and later two independent researchers Melter and Harary in 1976 renamed this parameter from locating set to resolving set [**40**]. After the applied study of this topic, in 2000 Chartrand again renamed this parameter for pure graph theoretical study and called as metric basis [**41**]. In [**42**] proposed the partition dimension of the graph as a version of the metric dimension of graphs in the year 2000. Now given below are the basic and necessary concepts of this study.

Definition 3.1. *In [4]*" Suppose $D(A(D), L(D))$ is an undirected graph of a chemical structure (network) with $A(D)$ is called as set of principal nodes (vertex set) and $L(D)$ is the set of branches (edge

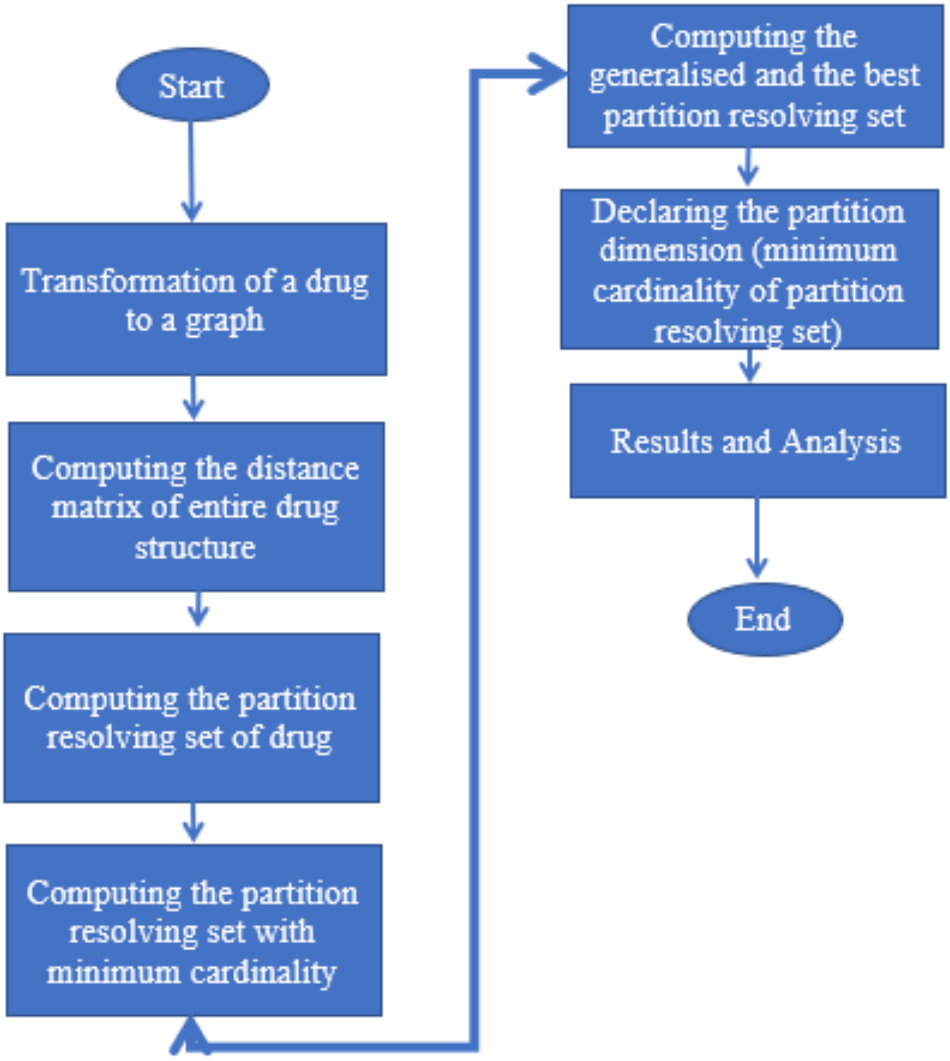

Fig. 1. Flowchart of Partition dimension

set). The distance between two principal nodes $a_1, a_2 \in A(D)$, denoted as $d(a_1, a_2)$ is the minimum count of branches between $a_1 - a_2$ path."

Definition 3.2. *In [4]* "Suppose $R \subset A(D)$ is the subset of principal nodes set and defined as $R = \{a_1, a_2, \ldots, a_s\}$, and let a principal node $a \in A(D)$. The identification or locations $r(a|R)$ of a principal node a with respect to R is actually a $s-$ordered distances $(d(a, a_1), d(a, a_2), \ldots, d(a, a_s))$. If each principal node from $A(D)$ have unique identification according to the ordered subset R, then this subset renamed as a resolving set of network D. The minimum numbers of the elements in the subset R is actually the metric dimension of D and it is denoted by the term $dim(D)$."

Definition 3.3. *In [4]* "Let $R_p \subseteq A(D)$ is the s-elements proper set and $r(a|R_p) = \{d(a, R_{p1}), d(a, R_{p2}), \ldots, d(a$ is the s-tuple distance identification of a principal node a in association with R_p. If the entire set of principal nodes have unique identifications, then R_p is named as the partition resolving set of the principal node of a network D. The least possible count of the subsets in that set of $A(D)$ is labeled as the partition dimension $(pd(D))$ of D."

For further understanding its flowchart given in the Figure 1.

Due to its vast applied look and usage, there are many abstract applications have also found since 1975, such as combinatorial optimization, robot roving, complicated games, image processing, phar-

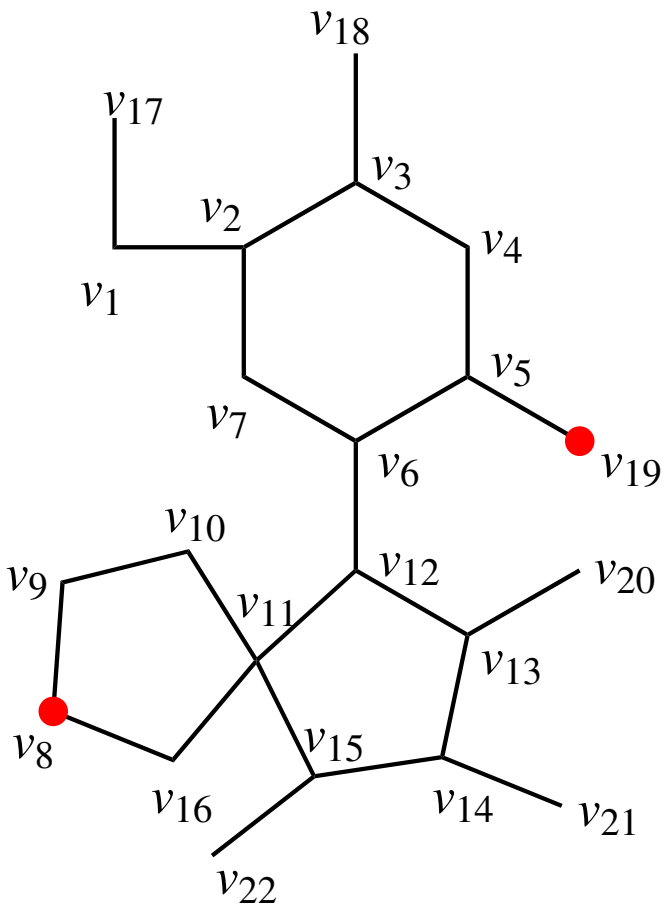

Fig. 2. Molecular graph of Amathaspiramide E with its first and second partition resolving subset and remaining vertices will be in its third subset

maceutical chemistry, the polymer sector, and the electric field are just a few areas where the metric dimension is used. One can find all of these applications in [**32**, **43**, **44**, **45**, **46**]. Robot roving is related to the idea of applications of a vertex set's partitioning in terms of metrics [**43**], while Djokovic Winkler relation [**47**], network verification and discovery, chemistry [**48**], mastermind games [**49**], image processing, pattern recognition, and hierarchical data structures are related to a structure's dimensions [**50**]. There are further applications in the literature [**51**, **33**].

4. Main Results: Anticancer Drugs and their Locating sets

We labeled the molecular graph of Amathaspiramide E drug with D_1. The Amathaspiramide E is an anticancer drug with molecular formula $C_{15}H_{16}Br_2N_2O_3$, and with molecular weight is 432.11. In graph theoretical study its order (or count of atoms) is $D_1 = 22$ and the size (or count of links) is $D_1 = 24$. The atom and links sets of D_1 are shown below. Furthermore, Figure 2, shows the molecular graph of Amathaspiramide E.

$$\begin{aligned} A(D_1) =& \{v_\zeta : 1 \leq \zeta \leq 22\}, \\ L(D_1) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 6, 8, 9, \ldots, 14\} \cup \{v_{11}v_{15}, v_{11}v_{16}, v_8v_{16}, v_6v_{12}, v_5v_{19}, \\ & v_{13}v_{20}, v_{14}v_{21}, v_{15}v_{22}, v_1v_{17}, v_3v_{18}, v_2v_7\} \end{aligned}$$

Theorem 4.1. *Let D_1 be a molecular graph of Amathaspiramide E anticancer drug. Then $pd(D_1) = 3$.*

Proof. To prove that $pd(D_1) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_8\}, R_{p2} = \{v_{19}\}$, and $R_{p3} = A(D_1) \setminus \{v_8, v_{19}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following representations of the entire vertex set of (D_1), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (8-\zeta, 6-\zeta, 0), & \text{if } \zeta = 1, 2; \\ (10-\zeta, 6-\zeta, 0), & \text{if } \zeta = 3, 4, 5; \\ (10-\zeta, \zeta-4, 0), & \text{if } \zeta = 6; \\ (5, \zeta-4, 0), & \text{if } \zeta = 7; \\ (i-8, 6, 1), & \text{if } \zeta = 8; \\ (\zeta-8, 15-\zeta, 0), & \text{if } \zeta = 9, 10; \\ (\zeta-9, 15_\zeta, 0), & \text{if } \zeta = 11, 12; \\ (\zeta-9, 4, 0), & \text{if } \zeta = 13; \\ (18-\zeta, 5, 0), & \text{if } \zeta = 14, 15, 16; \\ (8, 6, 0), & \text{if } \zeta = 17; \\ (8, 4, 0), & \text{if } \zeta = 18; \\ (25-\zeta, 0, 1), & \text{if } \zeta = 19; \\ (25-\zeta, 5, 0), & \text{if } \zeta = 20; \\ (26-\zeta, 6, 0), & \text{if } \zeta = 21, 22; \end{cases}$$

As a result, it follows in the form of a representation from the reasons above that $pd(D_1) \leq 3$ because all the vertices of (D_1) have the unique representations concerning partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_1) > 3$, which is $pd(D_1) = 2$. It is not true, because the partition dimension two is only possible for the path graph and chosen graph is not a path graph. So, $pd(D_1) = 3$, which is required result. □

We labeled the molecular graph of Carmustine drug with D_2. The Carmustine is an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_2 = 12$ and the size (or count of links) is $D_2 = 11$. The atom and links sets of D_2 are shown below. Furthermore, Figure 3, shows the molecular graph of Carmustine.

$$\begin{aligned} A(D_2) =& \{v_\zeta : 1 \leq \zeta \leq 12\}, \\ L(D_2) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 8, 10\} \cup \{v_5 v_{12}, v_4 v_{10}\}. \end{aligned}$$

Theorem 4.2. *Let D_2 be a molecular graph of Carmustine anticancer drug. Then $pd(D_2) = 3$.*

Proof. To prove that $pd(D_2) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_3\}, R_{p2} = \{v_{12}\}$, and $R_{p3} = A(D_2) \setminus \{v_3, v_{12}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following rep-

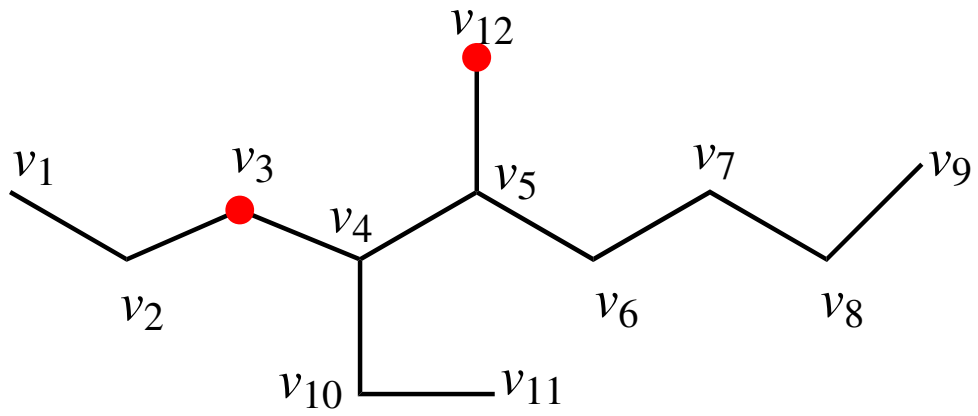

Fig. 3. Molecular graph of Carmustine with its first and second partition resolving subset and remaining vertices will be in its third subset

resentations of the entire vertex set of (D_2), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (3-\zeta, 6-\zeta, z_1)\,, & \text{if } \zeta = 1,2,3; \\ (\zeta-3, 6-\zeta, 0)\,, & \text{if } \zeta = 4,5; \\ (\zeta-3, \zeta-4, 0)\,, & \text{if } \zeta = 7,8,9; \\ (2, \zeta-7, 0)\,, & \text{if } \zeta = 10; \\ (3, \zeta-7, 0)\,, & \text{if } \zeta = 11; \\ (3, 0, 1)\,, & \text{if } \zeta = 12; \end{cases}$$

where $z_1 = \begin{cases} 1, & \text{if } \zeta = 3; \\ 0, & \text{otherwise.} \end{cases}$

As a result, it follows in the form of a representation from the reasons above that $pd(D_2) \leq 3$ because all the vertices of (D_2) have the unique representations to partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_2) > 3$, which is $pd(D_2) = 2$. It is not true, because the metric dimension two is only possible for the path graph and the chosen graph is not a path graph. So, $pd(D_2) = 3$, which is required result. □

We labelled the molecular graph of Caulibugulone E drug with D_3. The Caulibugulone E is an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_3 = 14$, and the size (or count of links) is $D_3 = 15$. The atom and links sets of D_3 are shown below. Furthermore, Figure 4, shows the molecular graph of Caulibugulone E.

$$\begin{aligned} A(D_3) =& \{v_\zeta : 1 \leq \zeta \leq 14\}, \\ L(D_3) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 11\} \cup \{v_4v_{14}, v_3v_{12}, v_5v_{10}, v_{11}v_{13}\}. \end{aligned}$$

Theorem 4.3. *Let D_3 be a molecular graph of Caulibugulone E anticancer drug. Then $pd(D_3) = 3$.*

Proof. To prove that $pd(D_3) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_1\}, R_{p2} = \{v_{13}\}$, and $R_{p3} = A(D_3) \setminus \{v_1, v_{13}\}$. Using the Definition 3.1 to see the shortest

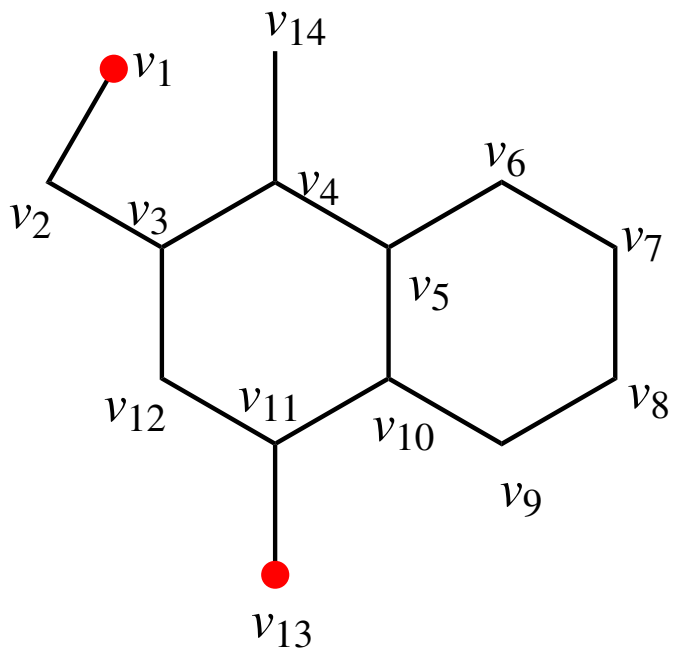

Fig. 4. Molecular graph of Caulibugulone E with its first and second partition resolving subset and remaining vertices will be in its third subset

distances and then arranging in the form given in the Definition 3.3, we constructed the following representations of the entire vertex set of (D_3), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R2_p), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (\zeta-1, 6-\zeta, z_2)\,, & \text{if } \zeta = 1,2,3; \\ (\zeta-1, 4, 0)\,, & \text{if } \zeta = 4; \\ (\zeta-1, \zeta-2, 0)\,, & \text{if } \zeta = 5,6,7; \\ (\zeta-1, 12-\zeta, 0)\,, & \text{if } \zeta = 8; \\ (15-\zeta, 12-\zeta, 0)\,, & \text{if } \zeta = 9,10,11; \\ (15-\zeta, 2, 0)\,, & \text{if } \zeta = 12; \\ (18-\zeta, 0, 1)\,, & \text{if } \zeta = 13; \\ (18-\zeta, 5, 0)\,, & \text{if } \zeta = 14; \end{cases}$$

where $z_2 = \begin{cases} 1, & \text{if } \zeta = 1; \\ 0, & \text{otherwise.} \end{cases}$

As a result, it follows in the form of a representation from the reasons above that $pd(D_3) \leq 3$ because all the vertices of (D_3) have the unique representations with respect to partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_3) > 3$, which is $pd(D_3) = 2$. It is not true, because the metric dimension two is only possible for the path graph and the chosen graph is not a path graph. So, $pd(D_3) = 3$, which is required result. □

We labeled the molecular graph of the Aspidostomide E drug with D_4. The Aspidostomide E is one of an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_4 = 26$ and the size (or count of links) is $D_4 = 29$. The atom and links sets of D_4 are shown below. Furthermore, the

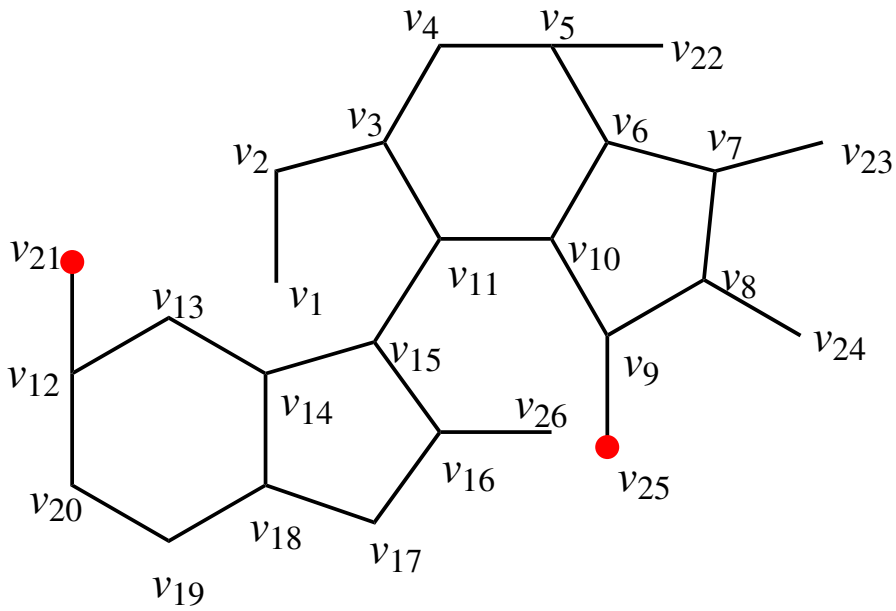

Fig. 5. Molecular graph of Aspidostomide E with its first and second partition resolving subset and remaining vertices will be in its third subset

Figure 5, shows the molecular graph of Aspidostomide E.

$$\begin{aligned} A(D_4) =& \{v_\zeta : 1 \leq \zeta \leq 26\}, \\ L(D_4) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 10, 12, 13, \ldots, 19\} \cup \{v_3v_{11}, v_5v_{22}, v_7v_{23}, v_8v_{24}, \\ & v_9v_{25}, v_6v_{10}, v_{11}v_{15}, v_{16}v_{26}, v_{14}v_{18}, v_{12}v_{21}, v_{12}v_{20}\}. \end{aligned}$$

Theorem 4.4. *Let D_4 be a molecular graph of Aspidostomide E anticancer drug. Then $pd(D_4) = 3$.*

Proof. To prove that $pd(D_4) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_{21}\}, R_{p2} = \{v_{25}\}$, and $R_{p3} = A(D_4) \setminus \{v_{21}, v_{25}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following rep-

resentations of the entire vertex set of (D_4), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (9-\zeta, 7-\zeta, 0), & \text{if } \zeta = 1, 2, 3; \\ (\zeta+3, 9-\zeta, 0), & \text{if } \zeta = 4, 5; \\ (\zeta+1, 9-\zeta, 0), & \text{if } \zeta = 6; \\ (\zeta+1, 10-\zeta, 0), & \text{if } \zeta = 7; \\ (16-\zeta, 10-\zeta, 0), & \text{if } \zeta = 8, 9; \\ (16-\zeta, \zeta-8, 0), & \text{if } \zeta = 10, 11; \\ (\zeta-11, 19-\zeta, 0), & \text{if } \zeta = 12, 13 \ldots 15; \\ (\zeta-11, \zeta-11, 0), & \text{if } \zeta = 16; \\ (22-\zeta, \zeta-11, 0), & \text{if } \zeta = 17; \\ (22-\zeta, \zeta-12, 0), & \text{if } \zeta = 18, 19, 20; \\ (0, 8, 1), & \text{if } \zeta = 21; \\ (9, 27-\zeta, 0), & \text{if } \zeta = 22, 23, 24; \\ (8, 0, 1), & \text{if } \zeta = 25; \\ (6, 6, 0), & \text{if } \zeta = 26; \end{cases}$$

As a result, it follows in the form of a representation from the reasons above that $pd(D_4) \leq 3$ because all the vertices of (D_4) have the unique representations with respect to partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_4) > 3$, which is $pd(D_4) = 2$. It is not true, because the metric dimension two is only possible for the path graph and the chosen graph is not a path graph. So, $dim(D_4) = 3$, which is required result. □

We labelled the molecular graph of Convolutamide A drug with D_5. The Convolutamide A is one of an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_5 = 31$ and the size (or count of links) is $D_5 = 32$. The atom and links sets of D_5 are shown below. Furthermore, Figure 6, shows the molecular graph of Convolutamide A.

$$\begin{aligned} A(D_5) =& \{v_\zeta : 1 \leq \zeta \leq 31\}, \\ L(D_5) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 5, 6, \ldots 10, 12, 13 \ldots, 24\} \cup \{v_1v_{30}, v_1v_6, v_6v_{29}, v_5v_{28}, v_3v_{11}, \\ & v_{11}v_{27}, v_{10}v_{26}, v_7v_{11}, v_9v_{12}, v_{12}v_{31}\}. \end{aligned}$$

Theorem 4.5. *Let D_5 be a molecular graph of Convolutamide A anticancer drug. Then $pd(D_5) = 3$.*

Proof. To prove that $pd(D_5) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_{11}\}, R_{p2} = \{v_{12}\}$, and $R_{p3} = A(D_5) \setminus \{v_{11}, v_{12}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following rep-

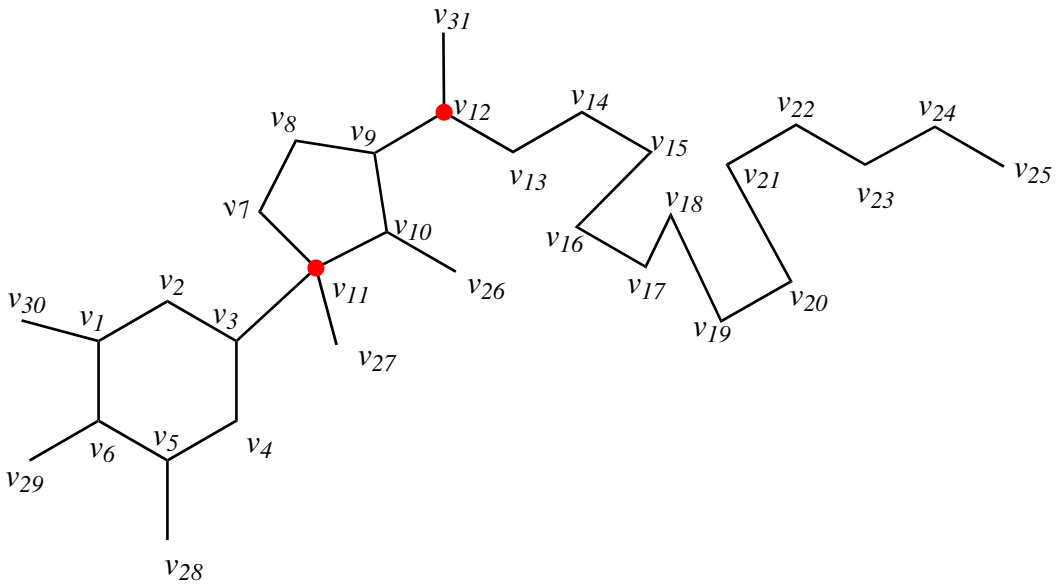

Fig. 6. Molecular graph of Convolutamide A with its first and second partition resolving subset and remaining vertices will be in its third subset

resentations of the entire vertex set of (D_5), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (3,\zeta)\,, & \text{if } \zeta = 1; \\ (6-\zeta,\zeta,0)\,, & \text{if } \zeta = 2,3,4; \\ (6-\zeta,8-\zeta,0)\,, & \text{if } \zeta = 5; \\ (\zeta-4,8-\zeta,0)\,, & \text{if } \zeta = 6; \\ (\zeta-4,\zeta-2,0)\,, & \text{if } \zeta = 7.8,\ldots,10; \\ (0,4,1)\,, & \text{if } \zeta = 11; \\ (4,0,1)\,, & \text{if } \zeta = 12; \\ (19-\zeta,17-\zeta,0)\,, & \text{if } \zeta = 13,14; \\ (19-\zeta,4,0)\,, & \text{if } \zeta = 15; \end{cases}$$

As a result, it follows in the form of a representation from the reasons above that $pd(D_5) \leq 3$ because all the vertices of (D_5) have the unique representations concerning partition resolving set R. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_5) > 3$, which is $dim(D_5) = 2$. It is not true, because the metric dimension two is only possible for the path graph and chosen graph is not a path graph. So, $pd(D_5) = 3$, which is required result. □

We labeled the molecular graph of the Convolutamine F drug with D_6. The Convolutamine F is one of an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_6 = 15$ and the size (or count of links) is $D_6 = 15$. The atom and links sets of D_6 are shown below. Furthermore, Figure 7, shows the molecular graph of Convolutamine F.

$$\begin{aligned} A(D_6) =& \{v_\zeta : 1 \leq \zeta \leq 15\}, \\ L(D_6) =& \{v_\zeta v_{\zeta+1} : \zeta = 1,2,\ldots,5,7,8,9,13\} \cup \{v_1v_{12}, v_2v_{14}, v_3v_{15}, \\ & v_4v_7, v_5v_{11}, v_1v_6\}. \end{aligned}$$

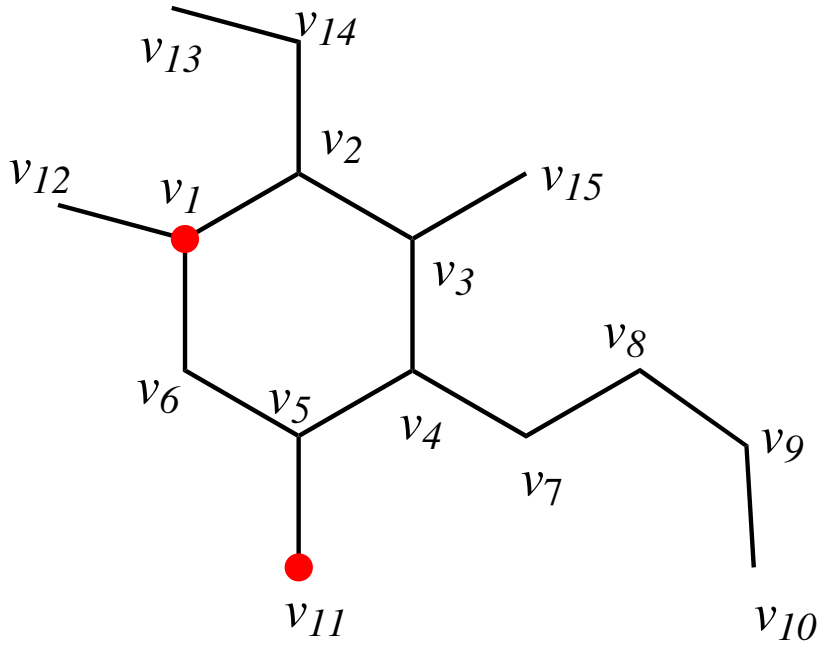

Fig. 7. Molecular graph of Convolutamine F with its first and second partition resolving subset and remaining vertices will be in its third subset

Theorem 4.6. *Let D_6 be a molecular graph of Convolutamine F anticancer drug. Then $pd(D_6) = 3$.*

Proof. To prove that $pd(D_6) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_1\}, R_{p2} = \{v_{11}\}$, and $R_{p3} = A(D_6) \setminus \{v_1, v_{11}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following representations of the entire vertex set of (D_6), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (6-\zeta, \zeta+1, z_3), & \text{if } \zeta = 1, 2, \ldots, 4; \\ (|\zeta-7|, \zeta+1, 0), & \text{if } \zeta = 5; \\ (|\zeta-7|, 13-\zeta, z_3), & \text{if } \zeta = 6, 7, \ldots, 13; \\ (5, 4, 0), & \text{if } \zeta = 14; \\ (18-\zeta, 6, 0), & \text{if } \zeta = 15; \\ (18-\zeta, 8, 0), & \text{if } \zeta = 16; \end{cases}$$

where $z_3 = \begin{cases} 1, & \text{if } \zeta = 1, 11; \\ 0, & \text{otherwise.} \end{cases}$

As a result, it follows in the form of a representation from the reasons above that $pd(D_6) \leq 3$ because all the vertices of (D_6) have the unique representations concerning partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_6) > 3$, which is $pd(D_6) = 2$. It is not true, because the metric dimension two is only possible for the path graph and the chosen graph is not a path graph. So, $pd(D_6) = 3$, which is required result. □

We labelled the molecular graph of Convolutamydine A drug with D_7. The Convolutamydine A is an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_7 = 16$, and the size (or count of links) is $D_7 = 18$. The atom and links sets of D_7 are shown below. Furthermore, Figure 8,

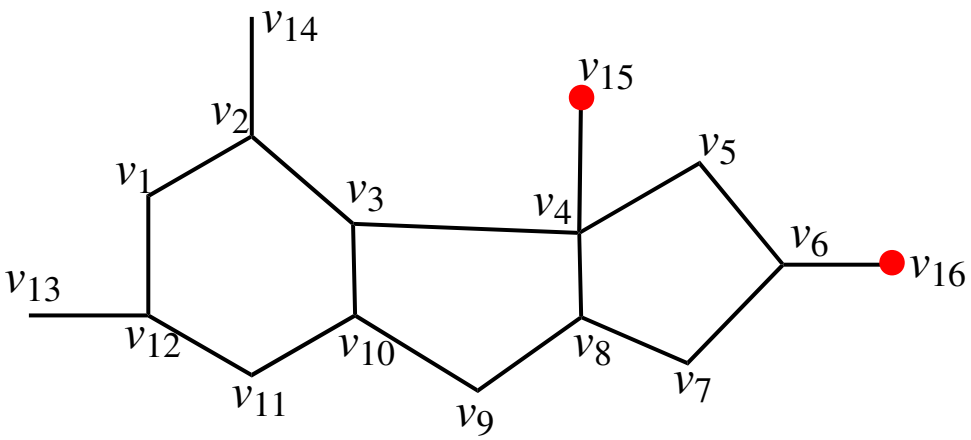

Fig. 8. Molecular graph of Convolutamydine A with its first and second partition resolving subset and remaining vertices will be in its third subset

shows the molecular graph of Convolutamydine A.

$$\begin{aligned} A(D_7) =& \{v_\zeta : 1 \leq \zeta \leq 16\}, \\ L(D_7) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 12\} \cup \{v_2v_{14}, v_4v_{15}, v_6v_{16}, \\ & v_3v_{10}, v_4v_8, v_1v_{12}\}. \end{aligned}$$

Theorem 4.7. *Let D_7 be a molecular graph of Convolutamydine A anticancer drug. Then $pd(D_7) = 3$.*

Proof. To prove that $pd(D_7) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_{15}\}, R_{p2} = \{v_{16}\}$, and $R_{p3} = A(D_7) \setminus \{v_{15}, v_{16}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following representations of the entire vertex set of (D_7), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (10-\zeta, 4-\zeta, 0), & \text{if } \zeta = 1, 2; \\ (10-\zeta, \zeta, 0), & \text{if } \zeta = 3, 4, 5; \\ (\zeta, \zeta, 0), & \text{if } \zeta = 6; \\ (\zeta, 13-\zeta, 0), & \text{if } \zeta = 7; \\ (\zeta-1, 13-\zeta, 0), & \text{if } \zeta = 8, 9; \\ (14-\zeta, \zeta-4, 0), & \text{if } \zeta = 10, 11, \ldots 13; \\ (2, \zeta-4, 0), & \text{if } \zeta = 14; \\ (0, 10, 1), & \text{if } \zeta = 15; \\ (\zeta-7, 16-\zeta, 1), & \text{if } \zeta = 16; \end{cases}$$

As a result, it follows in the form of a representation from the reasons above that $pd(D_7) \leq 3$ because all the vertices of (D_7) have the unique representations concerning partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_7) > 3$, which is $pd(D_7) = 2$. It is not true, because the metric dimension two is only possible for the path graph and chosen graph is not a path graph. So, $pd(D_7) = 3$, which is required result. □

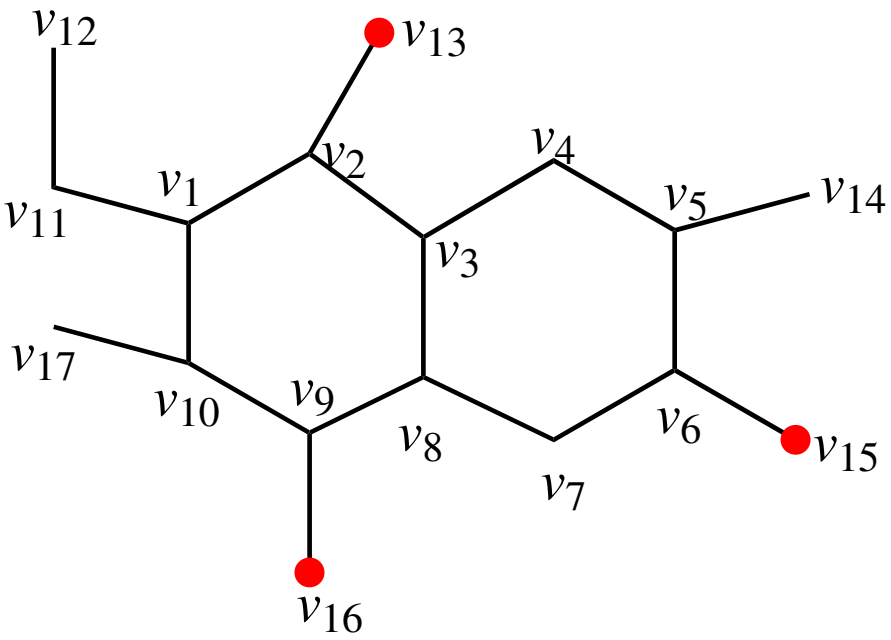

Fig. 9. Molecular graph of Perfragilin A with its first and second partition resolving subset and remaining vertices will be in its third subset

We labelled the molecular graph of Perfragilin A drug with D_8. The Perfragilin A is an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_8 = 17$, and the size (or count of links) is $D_8 = 18$. The atom and links sets of D_8 are shown below. Furthermore, Figure 9, shows the molecular graph of Perfragilin A.

$$\begin{aligned} A(D_8) =& \{v_\zeta : 1 \leq \zeta \leq 17\}, \\ L(D_8) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 9, 11\} \cup \{v_1v_{11}, v_1v_{10}, v_2v_{13}, \\ & v_5v_{14}, v_6v_{15}, v_9v_{16}, v_{10}v_{17}, v_3v_8\}. \end{aligned}$$

Theorem 4.8. *Let D_8 be a molecular graph of Perfragilin A anticancer drug. Then $pd(D_8) = 4$.*

Proof. To prove that $pd(D_8) = 4$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}, R_{p4}\}$, where $R_{p1} = \{v_{13}\}, R_{p2} = \{v_{15}\}, R_{p3} = \{v_{16}\}$, and $R_{p4} = A(D_8) \setminus \{v_{13}, v_{15}, v_{16}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following representations of the entire vertex set of (D_8), which is $r(v_\zeta|R_p) =$

$(d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}), d(v_\zeta|R_{p4}))$.

$$r(v_\zeta|R_p) = \begin{cases} (2, 7-\zeta, \zeta+2, 0), & \text{if } \zeta = 1; \\ (\zeta-1, 7-\zeta, \zeta+2, 0), & \text{if } \zeta = 2; \\ (\zeta-1, 7-\zeta, \zeta, 0), & \text{if } \zeta = 3, 4, 5; \\ (\zeta-1, 7-\zeta, 10-\zeta, 0), & \text{if } \zeta = 6; \\ (11-\zeta, \zeta-5, 10-\zeta, 0), & \text{if } \zeta = 7, 8; \\ (13-\zeta, \zeta-5, 10-\zeta, 0), & \text{if } \zeta = 9; \\ (13-\zeta, \zeta-5, 2, 0), & \text{if } \zeta = 10; \\ (\zeta-9, \zeta-4, \zeta-7, 0), & \text{if } \zeta = 11, 12; \\ (0, 6, \zeta-8, 1), & \text{if } \zeta = 13; \\ (5, 3\zeta-8, 0), & \text{if } \zeta = 14; \\ (21-\zeta, 0, 5, 1), & \text{if } \zeta = 15; \\ (21-\zeta, \zeta-11, 0, 1), & \text{if } \zeta = 16; \\ (21-\zeta, \zeta-11, 3, 0), & \text{if } \zeta = 17; \end{cases}$$

As a result, it follows in the form of a representation from the reasons above that $pd(D_8) \leq 4$ because all the vertices of (D_8) have the unique representations with respect to partition resolving set R_p.

Now we prove that $pd(D_8) \geq 4$. On contradictory, we assume $pd(D_8) = 3$. For this consider the resolving set R'_p with cardinality 3. Following discussion for this assumption.

Case: Let R'_p with cardinality 3. The same representation are; $r\left(v_{13}|R'_p\right) = r\left(v_{15}|R'_p\right)$. Similarly, by taking any possible subset from the atom set of Perfragilin A, the situation becomes: let $r(v_\alpha|R'_p) = r(v_\beta|R'_p)$ if and only if when $d(v_\alpha, v_\beta) = 2$. This concludes that $pd(D_8) = 4$. □

We labelled the molecular graph of Melatonin drug with D_9. Melatonin is one of an anticancer drug, in graph theoretical study its order (or count of atoms) is $D_9 = 17$ and the size (or count of links) is $D_9 = 18$. The atom and links sets of D_9 are shown below. Furthermore, Figure 10, shows the molecular graph of Melatonin.

$$\begin{aligned} A(D_9) =& \{v_\zeta : 1 \leq \zeta \leq 17\}, \\ L(D_9) =& \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 8, 9, \ldots, 13, 16\} \cup \{v_{13}v_{15}, v_4v_8, v_2v_{16}, v_1v_9, v_5v_{10}\}. \end{aligned}$$

Theorem 4.9. *Let D_9 be a molecular graph of Melatonin anticancer drug. Then $pd(D_9) = 3$.*

Proof. To prove that $pd(D_9) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_2\}, R_{p2} = \{v_{17}\}$, and $R_{p3} = A(D_9) \setminus \{v_2, v_{17}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following rep-

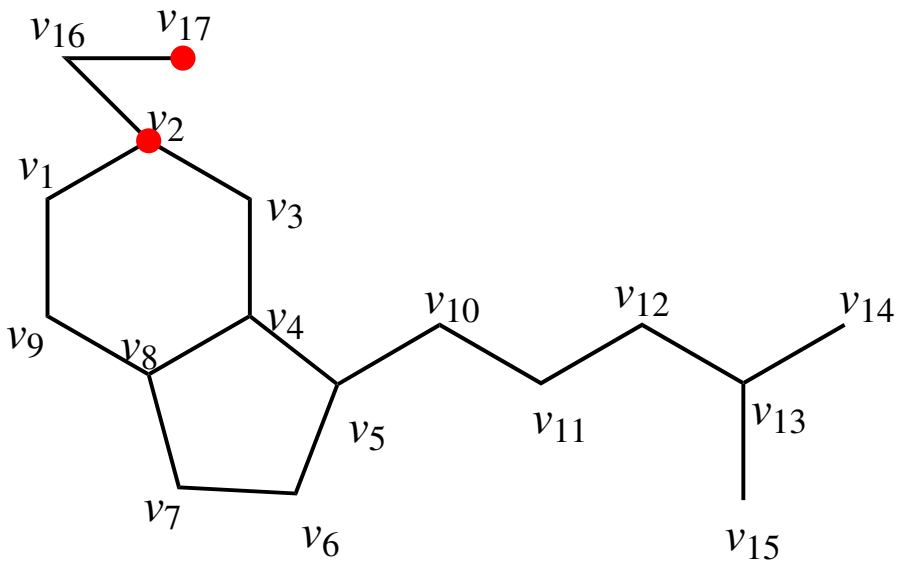

Fig. 10. Molecular graph of Melatonin with its first and second partition resolving subset and remaining vertices will be in its third subset

resentations of the entire vertex set of (D_9), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (|\zeta-2|, 12-\zeta, z_4)\,, & \text{if } \zeta = 1,2,3; \\ (2, 6+\zeta, 0)\,, & \text{if } \zeta = 4,5; \\ (\zeta-4, 8, 0)\,, & \text{if } \zeta = 6,7; \\ (\zeta-4, 15-\zeta, 0)\,, & \text{if } \zeta = 8; \\ (13-\zeta, 15-\zeta, 0)\,, & \text{if } \zeta = 9; \\ (13-\zeta, 7, 0)\,, & \text{if } \zeta = 10; \\ (\zeta-6, 16-\zeta, 0)\,, & \text{if } \zeta = 11, 12, \ldots 15; \\ (10, 2, 0)\,, & \text{if } \zeta = 16; \\ (10, 0, 1)\,, & \text{if } \zeta = 17; \\ (\zeta-13, \zeta-10, 0)\,, & \text{if } \zeta = 18, 19; \end{cases}$$

where $z_4 = \begin{cases} 1, & \text{if } \zeta = 2; \\ 0, & \text{otherwise.} \end{cases}$

As a result, it follows in the form of a representation from the reasons above that $pd(D_9) \leq 3$ because all the vertices of (D_9) have the unique representations with respect to partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_9) > 3$, which is $pd(D_9) = 2$. It is not true, because the metric dimension two is only possible for the path graph and chosen graph is not a path graph. So, $pd(D_9) = 3$, which is required result. □

We labelled the molecular graph of Tambjamine K drug with D_{10}. The Tambjamine K is an anti-cancer drug, in graph theoretical study its order (or count of atoms) is $D_{10} = 19$ and the size (or count of links) is $D_{10} = 20$. The atom and links sets of D_{10} are shown below. Furthermore, the Figure 11, shows the molecular graph of Tambjamine K.

$A(D_{10}) = \{v_\zeta : 1 \leq \zeta \leq 19\}$,
$L(D_{10}) = \{v_\zeta v_{\zeta+1} : \zeta = 1, 2, \ldots, 4, 6, 7, \ldots, 9, 11, 12, \ldots, 15, 18\} \cup \{v_1v_5, v_3v_6, v_6v_{10}, v_8v_{18}, v_9v_{11}, v_{15}v_{17}\}$.

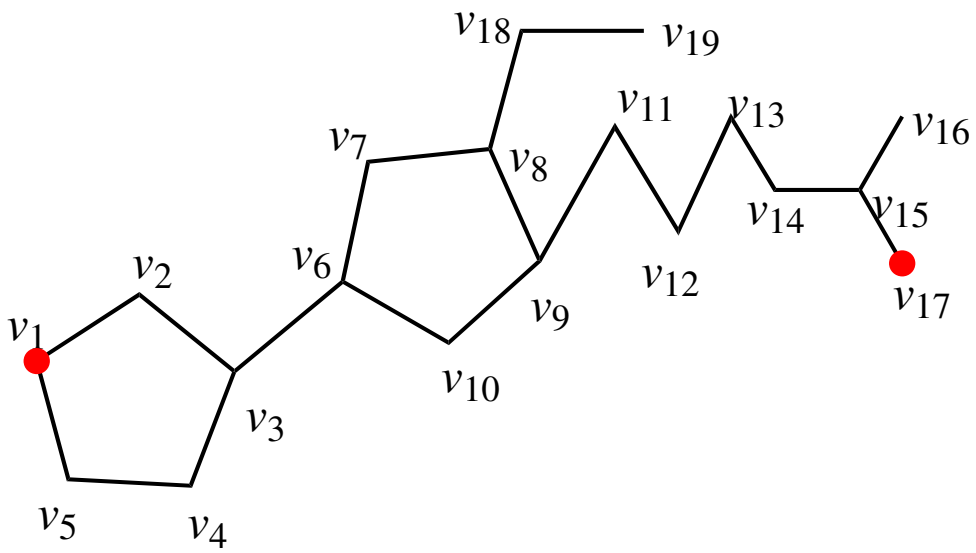

Fig. 11. Molecular graph of Tambjamine K with its first and second partition resolving subset and remaining vertices will be in its third subset

Theorem 4.10. *Let D_{10} be a molecular graph of Tambjamine K anticancer drug. Then $pd(D_{10}) = 3$.*

Proof. To prove that $pd(D_{10}) = 3$, assume a partition locating set $R_p = \{R_{p1}, R_{p2}, R_{p3}\}$, where $R_{p1} = \{v_1\}, R_{p2} = \{v_{17}\}$, and $R_{p3} = A(D_{10}) \setminus \{v_1, v_{17}\}$. Using the Definition 3.1 to see the shortest distances and then arranging in the form given in the Definition 3.3, we constructed the following representations of the entire vertex set of (D_{10}), which is $r(v_\zeta|R_p) = (d(v_\zeta|R_{p1}), d(v_\zeta|R_{p2}), d(v_\zeta|R_{p3}))$.

$$r(v_\zeta|R_p) = \begin{cases} (\zeta, 8-\zeta, z_5), & \text{if } \zeta = 1, 2, 3; \\ (\zeta, \zeta+2, 0), & \text{if } \zeta = 4; \\ (8-\zeta, \zeta+2, 0), & \text{if } \zeta = 5, 6; \\ (\zeta-2, 11-\zeta, 0), & \text{if } \zeta = 7, 8; \\ (15-\zeta, 11-\zeta, 0), & \text{if } \zeta = 9; \\ (15-\zeta, \zeta-7, 0), & \text{if } \zeta = 10, 11; \\ (\zeta-5, \zeta_1 1, z_5), & \text{if } \zeta = 12, 13, \ldots 25; \\ (32-\zeta, \zeta-22, 0), & \text{if } \zeta = 26, 27; \\ (32-\zeta, \zeta-20, 0), & \text{if } \zeta = 28, 29; \\ (0, 8, 0), & \text{if } \zeta = 30; \\ (8, 0, 0), & \text{if } \zeta = 31; \end{cases}$$

where $z_5 = \begin{cases} 1, & \text{if } \zeta = 1, 17; \\ 0, & \text{otherwise.} \end{cases}$

As a result, it follows in the form of a representation from the reasons above that $pd(D_{10}) \leq 3$ because all the vertices of (D_{10}) have the unique representations with respect to partition resolving set R_p. Now for the exactness instead of bound, we will follow the method of contradiction $pd(D_{10}) > 3$, which is $pd(D_{10}) = 2$. It is not true, because the metric dimension two is only possible for the path graph and chosen graph is not a path graph. So, $pd(D_{10}) = 3$, which is required result. □

5. Conclusion

To cure cancer disease anticancer medications are used. In this work, we studied some of the anticancer medications in terms of partition locating set, where partition locating set is an improper subset to settled entire atom set of a graph into a unique way to access each atom independently. Moreover, the summary of main results are given as: the Amathaspiramide E, Carmustine, Caulibugulone E, Aspidostomide E, Convolutamide A, Convolutamine F, Convolutamydine A, Melatonin, and Tambjamine K has three partition dimension while the only one drug which is Perfragilin A containing four elements in their partition resolving set.

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Conflicts of Interest

The authors declare that they have no conflicts of interest.

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