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Dynamic Aggregation Operators for Selection of Optimal Communication System in the Rescue Department Under Complex q-rung Orthopair Fuzzy Environment

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ABSTRACT

Selecting the advanced communication system for the rescue department is pivotal for enhancing coordination, operations, and effectiveness in today's dynamic environment. Addressing the complexities arising from uncertainty and periodicity, the Complex q-rung Orthopair Fuzzy Set theory emerges as adept, encapsulating comprehensive problem specifications. This study introduces two innovative aggregation operators within the Cq-ROFS framework: the Complex q-rung Orthopair Fuzzy Dynamic Weighted Averaging (Cq-ROFDWA) and the Complex q-rung Orthopair Fuzzy Dynamic Weighted Geometric (Cq-ROFDWG) operators. Some important characteristics of the newly defined operators are established. Moreover, these operators contribute to a systematic framework for handling Multiple Attribute Decision Making (MADM) problems involving complex q-rung Orthopair fuzzy information. The article exemplifies their application in resolving a MADM problem, determining the optimal option of an advanced communication system. Finally, to validate the derived methodologies, a thorough comparison study is carried out, demonstrating the superiority of the presented operators against various existing operators.

1. Introduction

To dispose unknown or undetermined information in the field of decision making, Zadeh [1] proposed the innovative concept of fuzzy set (FS) in 1965, which is characterized by a membership function limited to [0, 1], and it has been proven to be a very powerful tool to deal with uncertain information

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in real-life problems. Now there are many extensions of fuzzy sets such as. Although fuzzy sets offer numerous benefits, there are instances where it is challenging or unfeasible to resolve the problem solely using a membership function. To address this limitation, Atanassov [2] introduced the concept of intuitionistic fuzzy sets (IFS) as an extension of fuzzy sets (FS). IFS is characterized by a membership function, a non-membership function, and a degree of indeterminacy or uncertainty, all of which belong to the range [0, 1]. A key restriction of IFS is that the sum of the membership and non-membership degrees must be less than or equal to 1. The IFS concept provides a more robust framework than FS for tackling uncertain issues and offers a more effective means to represent human opinions. Additionally, this concept has gained significant interest and has been successfully applied across fields such as mathematics, engineering, information sciences, and multi-criteria decision-making (MCDM) problems [3–6]. Particularly in the context of multi-attribute decision-making (MADM) problems, intuitionistic fuzzy sets (IFS) have become a crucial method for addressing various decision-making challenges across different domains[7–11]. Yager [12] introduced the concept of Pythagorean fuzzy sets (PyFS), which is defined by the condition that the sum of the squares of the membership degree and the non-membership degree must be less than or equal to 1. Garg[13] introduced several novel generalized Pythagorean fuzzy information aggregation methods using Einstein operations and explored their applications in decision-making processes. Peng and Yang [14] introduced the Pythagorean fuzzy Choquet integral and the MABAC method, both of which are grounded in Pythagorean fuzzy information. For additional details regarding PyFSs, one may refer to [15, 16]. Yager [17]presented the idea of q-rung orthopair fuzzy sets (q-ROFS), which offer greater flexibility and generality compared to intuitionistic fuzzy sets (IFS) and Pythagorean fuzzy sets (PyFS). This method is intended to manage intricate and ambiguous data within the context of fuzzy set theory. Additionally, Liu and Wang[18] introduced q-ROF aggregation operators to average the assessment data. Nevertheless, many scholars raised the question of what would occur if the co-domain of fuzzy sets (FS) were altered to a set of complex numbers instead of the range [0, 1]. The response to this inquiry was provided by Ramot[19] in 2002, who defined the concept of complex fuzzy sets (CFS) as an extension of FS. The concept associated with CFSs has also been proposed in [20–22]. The development of the relationship between complex and fuzzy set theory led to the suggestion that the range of the membership function be confined to the complex unit circle. This modification enables complex fuzzy sets (CFS) to represent two-dimensional information, incorporating both amplitude and phase components. Both the amplitude and phase components are real-valued functions that can take values from the unit interval, capturing the uncertainty in both dimensions. By applying the concept of CFS, a variety of physical problems have been successfully addressed. This theory holds significant relevance in numerous fields, particularly in the prediction of periodic events and advanced control systems. These events involve multiple fuzzy variables that are interrelated in ways that traditional fuzzy operations cannot adequately handle. Dynamic aggregation operators on CIF were introduced in [23]. Akram and Naz [24] proposed an innovative decision-making approach based on CPFS theory. This set serves as an extension of both CFS and CIFS. The intricate fuzzy geometric aggregation operators and sophisticated fuzzy arithmetic aggregation operators were introduced in [25] and [26]. Garg and Rani [27] introduced CIF arithmetic and geometric aggregation operators. Within the framework of the CPFS environment, various operators, including Einstein geometric operators. For example, CqROFS theory is capable of managing two-dimensional data, whereas IFS, PFS, and FFS are unable to process such data. Additionally, the CIFS and CPFS theories do not account for situations where the sum of the squares of membership and non-membership degrees exceeds 1. In contrast, CqROFS can easily handle such cases. This research stands apart from prior studies due to its specific emphasis on incorporating time periods. It is essential to develop robust, efficient dynamic aggregation operators that can address decision-making challenges with evolving uncertainty, imprecision, and vagueness. By enabling the integration of information across different time periods, the proposed study provides a clearer and more accurate understanding of the issues. Therefore, it is crucial to conduct research focused on the challenges of dynamic complex fuzzy MADM. These factors drive us to propose dynamic operators within the CFF framework and offer a solution to the MADM problem. Dynamic aggregation operators are utilized to address uncertainties and imprecise information that change over time. Existing research on CFF is not equipped to evaluate time-dependent data. It is also crucial to recognize that without incorporating the time factor, shifts in the significance or value of various data points may not be accurately reflected in the aggregated results. This lack of responsiveness can lead to erroneous outcomes, especially in cases where the importance of data points fluctuates over time. Dynamic weighted aggregation operators are ideal for scenarios that require immediate decision-making. Their capacity to quickly adapt to changes in the environment or input data makes them particularly effective for time-sensitive decision processes. However, in numerous decision-making contexts, such as dynamic medical diagnostics, multi-period investment choices, the real-time evaluation of defense mechanisms, and ongoing personnel assessments, the key information needed for decisions is often gathered at different times.Our main goal is to develop and assess operators tailored to the distinct features of CFFS. These operators are specifically designed to improve flexibility and responsiveness in decision-making within complex and unpredictable situations. By incorporating these operators, the system gains the ability to dynamically modify membership functions and aggregation processes in response to changing conditions, fluctuating data, and evolving preferences of decision-makers. This adaptability is crucial in managing scenarios with varying degrees of uncertainty, making traditional static operators significantly less effective. Our study focuses on the following primary objectives within the theoretical framework:

1. Two novel aggregation operators, namely Cq-ROFDWA and Cq-ROFDWG operators, developed for decision making situation that incorporate complex q-rung orthopair fuzzy information.

2. Construct a dynamic operational system that is necessary for the execution of CQ-ROFSs. This involves the formation of mathematical model that decpite the relationships between different Cq-ROFNs.

3. Develop a systematic approach to address the MADM problem involving Cq-ROF data.

4. Conduct a comprehensive analysis of Cq-ROFDWA and Cq-ROFDWG operators, establishing their unique attributes and specific applications to foster a deeper comprehension of their capabilities.

Section 2 introduces several concepts, while the subsequent section 3 elaborates on various Cq-ROF operators. Segment 4 presents the development of the algorithm tackling MADM method. An example is demonstrated in Section 5, followed by comparative analysis in Section 7. The article concludes in Section 8, addressing limitations and offering suggestions for future study.

2. Some basic concepts

In this section, we defined fundamental principles associated with Complex q-rung orthopair fuzzy sets $(C_q - ROFS)$. [28] Let $Y = \{y_1, y_2, y_3, ..., y_n\}$ be any ordinary set, then a q-rung orthopair fuzzy set $(q - ROFS)\, Q$ defined on Y is expressed as follows:

$$Q = \{(y_i, (m_Q(y_i), n_Q(y_i))) \,|y_i \in Y\}, \tag{1}$$

where $m_Q(y_i) : Y \to [0, 1]$ and $n_Q(y_i) : Y \to [0, 1]$ denote membership and non-membership degree of the element $y \in Y$ to set Q, satisfying $0 \leq (m_Q(y_i))^q + (n_Q(y_i))^q \leq 1$ but $(q \geq 1)$. [29]

The degree of indeterminacy of element to set Q is expressed as

$$\Pi_Q(y_i) = (1 - (m_Q(y_i))^q + (n_Q(y_i))^q)^{\frac{1}{q}}. \quad (2)$$

Obviously $0 \leq \Pi_Q \leq 1, (y_i \in Y)$. For convenience $Q = (m_Q, n_Q)$ is called as a q-rung orthopair fuzzy number $(q - ROFN)$ which van be denoted as $a = (m, n)$. [29] Let $Y = \{y_1, y_2, y_3, ..., y_n\}$ be any ordinary set, then a complex q-rung orthopair fuzzy set $(Cq - ROFS)\, C$ defined on Y is expressed as follows:

$$C = \{y_i, (m'_C(y_i), n'_C(y_i)) \,|y_i \in Y\}, q \geq 1 \quad (3)$$

where $m'_C(y_i) = m(y)\, e^{i2\pi u_{m(y)}}$ and $n'_C(y_i) = n(y)\, e^{i2\pi u_{n(y)}}$ denote the complex-valued membership and non-membership degrees of the element $y \in Y$ to the set C, satisfying $0 \leq (m(y))^q + (n(y))^q \leq 1$ and $0 \leq (n(y))^q + (n(y))^q \leq 1$ where $m(y)$ and $u_m(y)$ represent the real and imaginary parts of the complex-valued membership degree, $n(y)$ and $u_n(y)$ represent the real and imaginary parts of the complex-valued non-membership degree. [30] Degree of indeterminacy of set C is expressed as follows:

$$I(y) = (1 - (m(y))^q - (n(y))^q)^{\frac{1}{q}}\, e^{i2\pi\left(1-\left(u_{m(y)}\right)^q-\left(u_{n(y)}\right)^q\right)^{\frac{1}{q}}}. \quad (4)$$

A complex q-rung orthopair fuzzy number $(Cq - ROFN)$ can be defined as:

$$\mathcal{C} = \left(m(y)\, e^{i2\pi u_{m(y)}}, n(y)\, e^{i2\pi u_{n(y)}}\right). \quad (5)$$

It can also be denoted as :

$$\mathcal{C} = \left(me^{i2\pi u_{m(y)}}, ne^{i2\pi u_{n(y)}}\right).$$

[31] For two Cq-ROFN, $A = (m_A e^{i2\pi u_{m_A}}, n_A e^{i2\pi u_{n_A}})$ and $B = (m_B e^{i2\pi u_{m_B}}, n_B e^{i2\pi u_{n_B}})$ then,

1. $A \subseteq B$ if $m_A \leq m_B, n_A \geq n_B$ and $u_{m_A} \leq u_{m_B}, u_{n_A} \geq u_{n_B}$,
2. A=B iff $m_A = m_B, n_A = n_B$ and $u_{m_A} = u_{m_B}, u_{n_A} = u_{n_B}$,
3. $A^c = (n_A e^{i2\pi u_{n_A}}, m_A e^{i2\pi u_{m_A}})$.

[32] Score function S and accuracy function A on a Cq-ROFN, $A = (m_A e^{i2\pi u_{m_A}}, n_A e^{i2\pi u_{n_A}})$ are defined as:

$$S(A) = \frac{1}{2}\left((m_A^q - n_A^q) + \left(u_{m_A}^q - u_{n_A}^q\right)\right), \quad (6)$$

$$A(A) = \frac{1}{2}\left((m_A^q + n_A^q) + \left(u_{m_A}^q + u_{n_A}^q\right)\right). \quad (7)$$

where $S(A), A(A) \in [-1, 1]$. An order relation between two Cq-ROFNs A and B is of the form:

1. If $S(A) > S(B)$ then $A > B$,
2. If $S(A) = S(B)$ then
   (a) If $A(A) > A(B)$ then $A > B$;
   (b) If $A(A) = A(B)$ then $A = B$.

For two Cq-ROFNs, $A = (m_A e^{i2\pi u_{m_A}}, n_A e^{i2\pi u_{n_A}})$, $B = (m_B e^{i2\pi u_{m_B}}, n_B e^{i2\pi u_{n_B}})$ and $\lambda > 0$ be any real number,

$A^c = (n_A e^{i2\pi u_{n_A}}, m_A e^{i2\pi u_{m_A}})$,

$$A \cup B = \left(max\left(m_A, m_B\right).e^{i2\pi max\left(u_{m_A}, u_{m_B}\right)}, min\left(n_A, n_B\right).e^{i2\pi min\left(u_{n_A}, u_{n_B}\right)} \right),$$

$$A \cap B = \left(min\left(m_A, m_B\right).e^{i2\pi min\left(u_{m_A}, u_{m_B}\right)}, max\left(n_A, n_B\right).e^{i2\pi max\left(u_{n_A}, u_{n_B}\right)} \right).$$

[33] Let $\sigma_k = \left(\left(m_k, e^{i2\pi u_{m_k}}\right), \left(n_k, e^{i2\pi u_{n_k}}\right)\right)$ for k=1,2,3,...,n be a collection of n Cq-ROFNs and let $w = [w_1, w_2, ..., w_n]^T$ represent the weight vector associated with σ_k such that, $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$. Then a Cq-ROF weighted averaging (Cq-ROFWA) operator is a function $Cq - ROFWA : \sigma^n \to \sigma$, defined as:

$$Cq - ROFWA\left(\sigma_1, \sigma_2, ..., \sigma_n\right) = \left(\left(\left(1 - \prod_{k=1}^{n} \left(1 - m_k^q\right)^{w_k}\right)^{\frac{1}{q}} .e^{i2\pi\left(1 - \prod_{k=1}^{n}\left(1 - u_{m_k}^q\right)^{w_k}\right)^{\frac{1}{q}}} \right), \prod_{k=1}^{n} n_k^{w_k}.e^{i2\pi\left(\prod_{k=1}^{n} u_{n_k}^{w_k}\right)} \right). \quad (8)$$

Let $\sigma_k = \left(\left(m_k, e^{i2\pi u_{m_k}}\right), \left(n_k, e^{i2\pi u_{n_k}}\right)\right)$ for k=1,2,3,...,n be a collection of n Cq-ROFNs and let $w = [w_1, w_2, ..., w_n]^T$ represent the weight vector associated with σ_k such that, $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$. Then a Cq-ROF weighted geometric (Cq-ROFWG) operator is a function $Cq - ROFWA : \sigma^n \to \sigma$, defined as:

$$Cq - ROFWG\left(\sigma_1, \sigma_2, ..., \sigma_n\right) = \left(\prod_{k=1}^{n} m_k^{w_k}.e^{i2\pi\left(\prod_{k=1}^{n} u_{m_k}^{w_k}\right)^{\frac{1}{q}}}, \left(1 - \prod_{k=1}^{n}\left(1 - n_k^q\right)^{w_k}\right)^{\frac{1}{q}} .e^{i2\pi\left(1 - \prod_{k=1}^{n}\left(1 - u_{n_k}^q\right)^{w_k}\right)^{\frac{1}{q}}} \right). \quad (9)$$

3. Dynamic aggregation operators on Complex q-rung orthopair fuzzy numbers

In this section, our aim is to develop dynamic aggregation operators in Cq-ROF environment.

[34] Let t denote the time variable. A Cq-ROF variable is represented as $\sigma_t = \left(\left(m_t, e^{i2\pi u_{m_t}}\right), \left(n_t, e^{i2\pi u_{n_t}}\right)\right)$, where $m_t, u_{m_t}, n_t, u_{n_t} \in [0,1]$, subject to the conditions $0 \leq m_t^q + n_t^q \leq 1$ and $0 \leq u_{m_t}^q + u_{n_t}^q \leq 1$. For Cq-ROF variable σ_t if $t = (t_1, t_2, ..., t_s)$ then $\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}$ indicate s Cq-ROFNs collected at s different periods. Consider two Cq-ROFNs, $\sigma_{t_1} = \left(\left(m_{t_1}, e^{i2\pi u_{m_{t_1}}}\right), \left(n_{t_1}, e^{i2\pi u_{n_{t_1}}}\right)\right)$ and $\sigma_{t_2} = \left(\left(m_{t_2}, e^{i2\pi u_{m_{t_2}}}\right), \left(n_{t_2}, e^{i2\pi u_{n_{t_2}}}\right)\right)$. The essential ordering principal governing their interactions as follows:

$\sigma_{t_1} \leq \sigma_{t_2}$ if $m_{t_1} \leq m_{t_2}, n_{t_1} \geq m_{t_2}$ and $u_{m_{t_1}} \leq u_{m_{t_2}}, u_{n_{t_1}} \geq u_{n_{t_2}}$ and $\sigma_{t_1} = \sigma_{t_2}$ if and only if $\sigma_{t_1} \subseteq \sigma_{t_2}$ and $\sigma_{t_2} \subseteq \sigma_{t_1}$. Let $\sigma_t = \left(\left(m_t, e^{i2\pi u_{m_t}}\right), \left(n_t, e^{i2\pi u_{n_t}}\right)\right), \sigma_{t_1} = \left(\left(m_{t_1}, e^{i2\pi u_{m_{t_1}}}\right), \left(n_{t_1}, e^{i2\pi u_{n_{t_1}}}\right)\right)$ and $\sigma_{t_2} = \left(\left(m_{t_2}, e^{i2\pi u_{m_{t_2}}}\right), \left(n_{t_2}, e^{i2\pi u_{n_{t_2}}}\right)\right)$ be three Cq-ROFNs and $\gamma_t > 0$. The dynamic operational laws for these Cq-ROFNs are defined as follows:

- $\sigma_{t_1} \oplus \sigma_{t_2} = \left(\left(\sqrt[q]{m_{t_1}^q + m_{t_2}^q - m_{t_1}^q m_{t_2}^q}, e^{i2\pi \sqrt[q]{u_{m_{t1}}^q + u_{m_{t2}}^q - u_{m_{t1}}^q u_{m_{t2}}^q}}\right), \left(n_{t_1} n_{t_2}, e^{i2\pi u_{n_{t1}} u_{n_{t2}}}\right)\right)$,
- $\sigma_{t_1} \times \sigma_{t_2} = \left(\left(m_{t_1} m_{t_2}, e^{i2\pi u_{m_{t1}} u_{m_{t2}}}\right), \left(\sqrt[q]{n_{t_1}^q + n_{t_2}^q - n_{t_1}^q n_{t_2}^q}, e^{i2\pi \sqrt[q]{u_{n_{t1}}^q + u_{n_{t2}}^q - u_{n_{t1}}^q u_{n_{t2}}^q}}\right)\right)$,

- $\sigma_t^{\gamma_t} = \left(\left(m_t^{\gamma_t}, e^{i2\pi u_{m_t}^{\gamma_t}}\right), \left(\sqrt[q]{1-(1-n_t^q)^{\gamma_t}}\right), e^{i2\pi\sqrt[q]{1-\left(1-u_{n_t}^q\right)^{\gamma_t}}}\right).$

In the subsequent definition, we introduce Cq-ROFDWA operator. Let us consider a collection of Cq-ROFNs denoted as $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$ for k=1,2,...,s, at distinct time periods t_k. Furthermore, let $\gamma_{t_k} = [\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s}]^T$ be the weight vector that is linked to time period t_k, subject to the conditions $\gamma_{t_k} \in [0,1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$.The Cq-ROFDWA operator denoted as $Cq-ROFDWA : \sigma^s \to \sigma$ defined as follows:

$$Cq-ROFDWA\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) = \left(\left(\sqrt[q]{1-\prod_{k=1}^{s}\left(1-m_{t_k}^q\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u_{m_{t_k}}^q\right)^{\gamma_{t_k}}}}\right), \left(\prod_{k=1}^{s} n_{t_k}^{\gamma_{t_k}}, e^{\prod_{k=1}^{s} u_{n_{t_k}}^{\gamma_{t_k}}}\right)\right). \tag{10}$$

Let $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$ for k=1,2,...,s, present a collection of Cq-ROFNs at s different time periods t_k. Additionally consider the weight vector with s time period $\gamma_{t_k} = [\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s}]^T$, where $\gamma_{t_k} \in [0,1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$. The aggregated value of these Cq-ROFNs using the Cq-ROFDWA operator is itself a Cq-ROFN, expressed as:

$$Cq-ROFDWA\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) = \left(\left(\sqrt[q]{1-\prod_{k=1}^{s}\left(1-m_{t_k}^q\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u_{m_{t_k}}^q\right)^{\gamma_{t_k}}}}\right), \left(\prod_{k=1}^{s} n_{t_k}^{\gamma_{t_k}}, e^{\prod_{k=1}^{s} u_{n_{t_k}}^{\gamma_{t_k}}}\right)\right).$$

The mathematical induction is employed to illustrate the proof of this result.
Let s=2;

$$Cq-ROFDWA\left(\sigma_{t_1}, \sigma_{t_2}\right) = \gamma_{t_1}.\sigma_{t_1} \oplus \gamma_{t_2}.\sigma_{t_2}$$

where;

$$\left(\gamma_{t_1}.\sigma_{t_1}\right) = \left(\left(\sqrt[q]{1-\left(1-m_{t_1}^q\right)^{\gamma_{t_1}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{m_{t_1}}^q\right)^{\gamma_{t_1}}}}\right), \left(n_{t_1}^{\gamma_{t_1}}, e^{i2\pi u_{n_{t_1}}^{\gamma_{t_1}}}\right)\right)$$

$$\left(\gamma_{t_2}.\sigma_{t_2}\right) = \left(\left(\sqrt[q]{1-\left(1-m_{t_2}^q\right)^{\gamma_{t_2}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{m_{t_2}}^q\right)^{\gamma_{t_2}}}}\right), \left(n_{t_2}^{\gamma_{t_2}}, e^{i2\pi u_{n_{t_2}}^{\gamma_{t_2}}}\right)\right)$$

Therefore

$$\left(\gamma_{t_1}.\sigma_{t_1}\right) \oplus \left(\gamma_{t_2}.\sigma_{t_2}\right) = \left(\left(\sqrt[q]{1-\left(1-m_{t_1}^q\right)^{\gamma_{t_1}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{m_{t_1}}^q\right)^{\gamma_{t_1}}}}\right), \left(n_{t_1}^{\gamma_{t_1}}, e^{i2\pi u_{n_{t_1}}^{\gamma_{t_1}}}\right)\right)$$

$$\oplus \left(\left(\sqrt[q]{1-\left(1-m_{t_2}^q\right)^{\gamma_{t_2}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{m_{t_2}}^q\right)^{\gamma_{t_2}}}}\right), \left(n_{t_2}^{\gamma_{t_2}}, e^{i2\pi u_{n_{t_2}}^{\gamma_{t_2}}}\right)\right)$$

$$= \left(\left(\sqrt[q]{1-\left(1-m_{t_1}^q\right)^{\gamma_{t_1}}\left(1-m_{t_2}^q\right)^{\gamma_{t_2}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{m_{t_1}}^q\right)^{\gamma_{t_1}}\left(1-u_{m_{t_2}}^q\right)^{\gamma_{t_2}}}}\right), \left(n_{t_1}^{\gamma_{t_1}} n_{t_2}^{\gamma_{t_2}}, e^{i2\pi u_{n_{t_1}}^{\gamma_{t_1}} u_{n_{t_2}}^{\gamma_{t_2}}}\right)\right)$$

Consequently,

$$Cq\text{−}ROFDWA\left(\sigma_{t_1},\sigma_{t_2}\right)=\left(\left(\sqrt[q]{1-\prod_{k=1}^{2}\left(1-m_{t_k}^{q}\right)^{\gamma_{t_k}}},e^{\sqrt[q]{1-\prod_{k=1}^{2}\left(1-u_{m_{t_k}}^{q}\right)^{\gamma_{t_k}}}}\right),\left(\prod_{k=1}^{2}n_{t_k}^{\gamma_{t_k}},e^{\prod_{k=1}^{2}u_{n_{t_k}}^{\gamma_{t_k}}}\right)\right).$$

Hence, the base case is proven for s=2.
Suppose that the statement holds for$s=r>2$, we can express it as follows

$$Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_r}\right)$$

$$=\oplus_{k=1}^{r}\left(\gamma_{t_k}.\sigma_{t_k}\right)=\left(\left(\sqrt[q]{1-\prod_{k=1}^{r}\left(1-m_{t_k}^{q}\right)^{\gamma_{t_k}}},e^{\sqrt[q]{1-\prod_{k=1}^{r}\left(1-u_{m_{t_k}}^{q}\right)^{\gamma_{t_k}}}}\right),\left(\prod_{k=1}^{r}n_{t_k}^{\gamma_{t_k}},e^{\prod_{k=1}^{r}u_{n_{t_k}}^{\gamma_{t_k}}}\right)\right).$$

Moreover, if s=r+1, then

$$Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_r},\sigma_{t_{r+1}}\right)=\gamma_{t_1}.\sigma_{t_1}\oplus\gamma_{t_2}.\sigma_{t_2}....\oplus\gamma_{t_r}.\sigma_{t_r}\oplus\gamma_{t_{r+1}}\sigma_{t_{r+1}}$$

$$=\left(\left(\sqrt[q]{1-\prod_{k=1}^{r+1}\left(1-m_{t_k}^{q}\right)^{\gamma_{t_k}}},e^{\sqrt[q]{1-\prod_{k=1}^{r+1}\left(1-u_{m_{t_k}}^{q}\right)^{\gamma_{t_k}}}}\right),\left(\prod_{k=1}^{r+1}n_{t_k}^{\gamma_{t_k}},e^{\prod_{k=1}^{r+1}u_{n_{t_k}}^{\gamma_{t_k}}}\right)\right).$$

Consequently, we may conclude that the claim is true for any positive integer s. The subsequent theorem investigates the idempotent characteristic of the Cq-ROFDWA operator. Consider a set of Cq-ROFNs $\sigma_{t_k}=\left(\left(m_{t_k},e^{i2\pi u_{m_{t_k}}}\right),\left(n_{t_k},e^{i2\pi u_{n_{t_k}}}\right)\right)$ for k=1,2,...,s. If $\sigma_{t_k}=\sigma_{t_l}$ for all k and some $l\in\{1,2,...,s\}$, where $\sigma_{t_l}=\left(\left(m_{t_l},e^{i2\pi u_{m_{t_l}}}\right),\left(n_{t_l},e^{i2\pi u_{n_{t_l}}}\right)\right)$ and $\gamma_{t_k}=[\gamma_{t_1},\gamma_{t_2},...,\gamma_{t_s}]^T$ represents the weight vector associated with time periods t_k, such that $\gamma_{t_k}\in[0,1]$ and $\sum_{k=1}^{s}\gamma_{t_k}=1$, then $Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right)=\sigma_{t_l}$.
Given that $\sigma_{t_k}=\sigma_{t_l}$ for all k=1,2,...,s and some $l\in\{1,2,...,s\}$ we can deduce that $m_{t_k}=m_{t_l},u_{m_{t_k}}=u_{m_{t_l}},n_{t_k}=n_{t_l}$ and $u_{n_{t_k}}=u_{n_{t_l}}$. Now let use delve into mathematical formulation of Cq-ROFDWA:

$$Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right)=$$

$$\left(\left(\sqrt[q]{1-\prod_{k=1}^{s}\left(1-m_{t_k}^{q}\right)^{\gamma_{t_k}}},e^{\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u_{m_{t_k}}^{q}\right)^{\gamma_{t_k}}}}\right),\left(\prod_{k=1}^{s}n_{t_k}^{\gamma_{t_k}},e^{\prod_{k=1}^{s}u_{n_{t_k}}^{\gamma_{t_k}}}\right)\right)$$

$$=\left(\left(\sqrt[q]{1-\left(1-m_{t_l}^{q}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}},e^{i2\pi\sqrt[q]{1-\left(1-u_{m_{t_l}}^{q}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}}}\right),\left(n_{t_l}^{\sum_{k=1}^{s}\gamma_{t_k}},e^{i2\pi u_{n_{t_l}}^{\sum_{k=1}^{s}\gamma_{t_k}}}\right)\right)$$

$$=\left(\left(\sqrt[q]{1-\left(1-m_{t_l}^{q}\right)},e^{i2\pi\sqrt[q]{1-\left(1-u_{m_{t_l}}^{q}\right)}}\right),\left(n_{t_l},e^{i2\pi u_{n_{t_l}}}\right)\right)$$

$$=\left(\left(\sqrt[q]{m_{t_l}^{q}},e^{i2\pi\sqrt[q]{u_{m_{t_l}}^{q}}}\right),\left(n_{t_l},e^{i2\pi u_{n_{t_l}}}\right)\right)$$

$$=\left(\left(m_{t_l},e^{i2\pi u_{m_{t_l}}}\right),\left(n_{t_l},e^{i2\pi u_{n_{t_l}}}\right)\right)$$

Consequently,

$$Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right)=\sigma_{t_l}.$$

The subsequent theorem investigates the boundedness characteristic of the Cq-ROFDWA operator. Let $\sigma_t^- = \left(\left(min\left\{m_{t_k}\right\}, min\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right), \left(max\left\{n_{t_k}\right\}, max\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)\right)$ and $\sigma_t^+ = \left(\left(max\left\{m_{t_k}\right\}, max\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right), \left(min\left\{n_{t_k}\right\}, min\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)\right)$ be the lower and upper bound of the Cq-ROFNs $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$ for k=1,2,...,s. Let $\gamma_{t_k} = (\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s})^T$ be the weight vector such that $\gamma_{t_k} \in [0,1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$. Then,

$$\sigma_t^- \leq Cq - ROFDWA\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) \leq \sigma_t^+.$$

Consider the result of applying the Cq-ROFDWA operator to the collection of Cq-ROFNs, denoted as

$$Cq - ROFDWA\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) = \left(\left(m_t, e^{i2\pi u_{m_t}}\right), \left(n_t, e^{i2\pi u_{n_t}}\right)\right)$$

For each

$$min\left\{m_{t_k}\right\} \leq m_{t_k} \leq max\left\{m_{t_k}\right\}$$

$$\implies min\left\{m_{t_k}^q\right\} \leq m_{t_k}^q \leq max\left\{m_{t_k}^q\right\}$$

$$\implies 1 - max\left\{m_{t_k}^q\right\} \geq 1 - m_{t_k}^q \geq 1 - min\left\{m_{t_k}^q\right\}$$

$$\implies \prod_{k=1}^{s}\left(1 - max\left\{m_{t_k}^q\right\}\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - m_{t_k}^q\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - min\left\{m_{t_k}^q\right\}\right)^{\gamma_{t_k}}$$

$$\implies \left(1 - max\left\{m_{t_k}^q\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - m_{t_k}^q\right)^{\gamma_{t_k}} \geq \left(1 - min\left\{m_{t_k}^q\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$\implies \left(1 - max\left\{m_{t_k}^q\right\}\right) \geq \prod_{k=1}^{s}\left(1 - m_{t_k}^q\right)^{\gamma_{t_k}} \geq \left(1 - min\left\{m_{t_k}^q\right\}\right)$$

$$\implies min\left\{m_{t_k}^q\right\} \leq 1 - \prod_{k=1}^{s}\left(1 - m_{t_k}^q\right)^{\gamma_{t_k}} \leq max\left\{m_{t_k}^q\right\}$$

$$\implies \sqrt[q]{min\left\{m_{t_k}^q\right\}} \leq \sqrt[q]{1 - \prod_{k=1}^{s}\left(1 - m_{t_k}^q\right)^{\gamma_{t_k}}} \leq \sqrt[q]{max\left\{m_{t_k}^q\right\}}.$$

So,

$$min\left\{m_{t_k}\right\} \leq m_t \leq max\left\{m_{t_k}\right\}$$

For each $e^{i2\pi u_{m_{t_k}}}$, we have

$$\implies min\left\{e^{i2\pi u_{m_{t_k}}^q}\right\} \leq e^{i2\pi u_{m_{t_k}}^q} \leq max\left\{e^{i2\pi u_{m_{t_k}}^q}\right\}$$

$$\implies 1 - max\left\{e^{i2\pi u_{m_{t_k}}^q}\right\} \geq 1 - e^{i2\pi u_{m_{t_k}}^q} \geq 1 - min\left\{e^{i2\pi u_{m_{t_k}}^q}\right\}$$

$$\implies \prod_{k=1}^{s}\left(1 - max\left\{e^{i2\pi u_{m_{t_k}}^q}\right\}\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - e^{i2\pi u_{m_{t_k}}^q}\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - min\left\{e^{i2\pi u_{m_{t_k}}^q}\right\}\right)^{\gamma_{t_k}}$$

$$\implies \left(1 - max\left\{e^{i2\pi u_{m_{t_k}}^q}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - e^{i2\pi u_{m_{t_k}}^q}\right)^{\gamma_{t_k}} \geq \left(1 - min\left\{e^{i2\pi u_{m_{t_k}}^q}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$\implies \left(1-max\left\{e^{i2\pi u^q_{m_{t_k}}}\right\}\right) \geq \prod_{k=1}^{s}\left(1-e^{i2\pi u^q_{m_{t_k}}}\right)^{\gamma_{t_k}} \geq \left(1-min\left\{e^{i2\pi u^q_{m_{t_k}}}\right\}\right)$$

$$\implies min\left\{e^{i2\pi u^q_{m_{t_k}}}\right\} \leq \prod_{k=1}^{s}\left(1-e^{i2\pi u^q_{m_{t_k}}}\right)^{\gamma_{t_k}} \leq max\left\{e^{i2\pi u^q_{m_{t_k}}}\right\}$$

$$\implies \sqrt[q]{min\left\{e^{i2\pi u^q_{m_{t_k}}}\right\}} \leq \sqrt[q]{\prod_{k=1}^{s}\left(1-e^{i2\pi u^q_{m_{t_k}}}\right)^{\gamma_{t_k}}} \leq \sqrt[q]{max\left\{e^{i2\pi u^q_{m_{t_k}}}\right\}}$$

Therefore,

$$min\left\{e^{i2\pi u_{m_{t_k}}}\right\} \leq e^{i2\pi u_{m_t}} \leq max\left\{e^{i2\pi u_{m_{t_k}}}\right\}$$

Moreover,

$$min\left\{n_{t_k}\right\} \leq n_{t_k} \leq max\left\{n_{t_k}\right\}$$

$$\implies \prod_{k=1}^{s}\left(min\left\{n_{t_k}\right\}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(n_{t_k}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(max\left\{n_{t_k}\right\}\right)^{\gamma_{t_k}}$$

$$\implies \left(min\left\{n_{t_k}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(n_{t_k}\right)^{\gamma_{t_k}} \leq \left(max\left\{n_{t_k}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$min\left\{n_{t_k}\right\} \leq n_{t_k} \leq max\left\{n_{t_k}\right\}$$

Furthermore

$$min\left\{e^{i2\pi u_{n_{t_k}}}\right\} \leq e^{i2\pi u_{n_{t_k}}} \leq max\left\{e^{i2\pi u_{n_{t_k}}}\right\}$$

$$\implies \prod_{k=1}^{s}\left(min\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(e^{i2\pi u_{n_{t_k}}}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(max\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)^{\gamma_{t_k}}$$

$$\implies \left(min\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(e^{i2\pi u_{n_{t_k}}}\right)^{\gamma_{t_k}} \leq \left(max\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$\implies min\left\{e^{i2\pi u_{n_{t_k}}}\right\} \leq e^{i2\pi u_{n_{t_k}}} \leq max\left\{e^{i2\pi u_{n_{t_k}}}\right\}$$

Hence by applying Definition 15, we obtain that

$$\sigma_t^- \leq Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right) \leq \sigma_t^+.$$

The subsequent theorem investigates the monotonic characteristics of the Cq-ROFDWA operator. Consider sets of two Cq-ROFNs, denoted as $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$ and $\sigma'_{t_k} = \left(\left(m'_{t_k}, e^{i2\pi u'_{m_{t_k}}}\right), \left(n'_{t_k}, e^{i2\pi u'_{n_{t_k}}}\right)\right)$ for k=1,2,...,s. Let $\gamma_{t_k} = \left[\gamma_{t_1},\gamma_{t_2},...,\gamma_{t_s}\right]^T$ represents the weight vector associated with time periods t_k, such that $\gamma_{t_k} \in [0,1]$ and $\sum_{k=1}^{s}\gamma_{t_k} = 1$. If for each t_k: $m_{t_k} \leq m'_{t_k}, e^{i2\pi u_{m_{t_k}}} \leq e^{i2\pi u'_{m_{t_k}}}$, and $n_{t_k} \geq n'_{t_k}, e^{i2\pi u_{n_{t_k}}} \geq e^{i2\pi u'_{n_{t_k}}}$ Then,

$$Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right) \leq Cq-ROFDWA\left(\sigma'_{t_1},\sigma'_{t_2},...,\sigma'_{t_s}\right).$$

Based on the provided description $\sigma_{t_k} and \sigma'_{t_k}$, we have

$$Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right) = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$$

and

$$Cq-ROFDWA\left(\sigma'_{t_1},\sigma'_{t_2},...,\sigma'_{t_s}\right)=\left(\left(m'_{t_k},e^{i2\pi u'_{m_{t_k}}}\right),\left(n'_{t_k},e^{i2\pi u'_{n_{t_k}}}\right)\right)$$

Since $m_{t_k}\leq m'_{t_k}$, which implies that ${m_{t_k}}^q\leq m'^q_{t_k}$, we can deduce that

$$1-m^q_{t_k}\geq 1-m'^q_{t_k}$$

$$\implies \prod_{k=1}^{s}\left(1-m^q_{t_k}\right)^{\gamma_{t_k}}\geq \prod_{k=1}^{s}\left(1-m'^q_{t_k}\right)^{\gamma_{t_k}}$$

$$\implies 1-\prod_{k=1}^{s}\left(1-m^q_{t_k}\right)^{\gamma_{t_k}}\leq 1-\prod_{k=1}^{s}\left(1-m'^q_{t_k}\right)^{\gamma_{t_k}}$$

$$\implies \sqrt[q]{1-\prod_{k=1}^{s}\left(1-m^q_{t_k}\right)^{\gamma_{t_k}}}\leq \sqrt[q]{1-\prod_{k=1}^{s}\left(1-m'^q_{t_k}\right)^{\gamma_{t_k}}}$$

Hence we can deduce that

$$m_{t_k}\leq m'_{t_k}\implies m_t\leq m'_t$$

Similarly, by considering $e^{i2\pi u_{m_{t_k}}}\leq e^{i2\pi u'_{m_{t_k}}}$ we derive

$$\implies e^{i2\pi\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u^q_{m_{t_k}}\right)^{\gamma_{t_k}}}}\leq e^{i2\pi\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u'^q_{m_{t_k}}\right)^{\gamma_{t_k}}}}$$

$$e^{i2\pi u_{m_t}}\leq e^{i2\pi u'_{m_t}}$$

By adopting the above procedure, we obtain the following inequalities,

$$n_t\geq n'_t$$

and

$$e^{i2\pi u_t}\geq e^{i2\pi u'_t}$$

Therefore, utilizing Definition 15 , we obtain that

$$Cq-ROFDWA\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right)\leq Cq-ROFDWA\left(\sigma'_{t_1},\sigma'_{t_2},...,\sigma'_{t_s}\right).$$

Thus, the monotonicity property is established.
In the ensuing definition, we present a dynamic geometric aggregation operators developed for the Cq-ROFNs, namely Cq-ROFDWG operator. Let us consider a collection of Cq-ROFNs denoted as $\sigma_{t_k}=\left(\left(m_{t_k},e^{i2\pi u_{m_{t_k}}}\right),\left(n_{t_k},e^{i2\pi u_{n_{t_k}}}\right)\right)$ for k=1,2,...,s, at distinct time periods t_k. Furthermore, let $\gamma_{t_k}=[\gamma_{t_1},\gamma_{t_2},...,\gamma_{t_s}]^T$ be the weight vector that is linked to time period t_k, subject to the conditions $\gamma_{t_k}\in[0,1]$ and $\sum_{k=1}^{s}\gamma_{t_k}=1$.The Cq-ROFWG operator denoted as $Cq-ROFDWG:\sigma^s\to\sigma$ defined as follows:

$$Cq-ROFDWG\left(\sigma_{t_1},\sigma_{t_2},...,\sigma_{t_s}\right)=$$
$$\left(\left(\prod_{k=1}^{s}m^{\gamma_{t_k}}_{t_k},e^{\prod_{k=1}^{s}u^{\gamma_{t_k}}_{m_{t_k}}}\right),\left(\sqrt[q]{1-\prod_{k=1}^{s}\left(1-n^q_{t_k}\right)^{\gamma_{t_k}}},e^{\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u^q_{n_{t_k}}\right)^{\gamma_{t_k}}}}\right)\right).$$

Let $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$ for k=1,2,...,s, at distinct time periods t_k. Furthermore, let $\gamma_{t_k} = [\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s}]^T$ be the weight vector that is linked to time period t_k, subject to the conditions $\gamma_{t_k} \in [0,1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$. The aggregated value of these Cq-ROFNs using the Cq-ROFDWG operator is itself a Cq-ROFN, expressed as follows:

$$Cq - ROFDWG(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}) = \left(\left(\prod_{k=1}^{s} m_{t_k}^{\gamma_{t_k}}, e^{\prod_{k=1}^{s} u_{m_{t_k}}^{\gamma_{t_k}}}\right), \left(\sqrt[q]{1 - \prod_{k=1}^{s}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u_{n_{t_k}}^q\right)^{\gamma_{t_k}}}}\right)\right).$$

We will use mathematical induction to prove this theorem . Let s=2, then

$$Cq - ROFDWG(\sigma_{t_1}, \sigma_{t_2}) = \sigma_{t_1}^{\gamma_{t_1}} \otimes \sigma_{t_2}^{\gamma_{t_2}}$$

Exploring the components,

$$\sigma_{t_1}^{\gamma_{t_1}} = \left(\left(m_{t_1}^{\gamma_{t_1}}, e^{i2\pi u_{m_{t_1}}^{\gamma_{t_1}}}\right), \left(\sqrt[q]{1 - \left(1 - n_{t_1}^q\right)^{\gamma_{t_1}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{n_{t_1}}^q\right)^{\gamma_{t_1}}}}\right)\right)$$

$$\sigma_{t_2}^{\gamma_{t_2}} = \left(\left(m_{t_2}^{\gamma_{t_2}}, e^{i2\pi u_{m_{t_2}}^{\gamma_{t_2}}}\right), \left(\sqrt[q]{1 - \left(1 - n_{t_2}^q\right)^{\gamma_{t_2}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{n_{t_2}}^q\right)^{\gamma_{t_2}}}}\right)\right)$$

Then,

$$\sigma_{t_1}^{\gamma_{t_1}} \otimes \sigma_{t_2}^{\gamma_{t_2}} = \left(\left(m_{t_1}^{\gamma_{t_1}}, e^{i2\pi u_{m_{t_1}}^{\gamma_{t_1}}}\right), \left(\sqrt[q]{1 - \left(1 - n_{t_1}^q\right)^{\gamma_{t_1}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{n_{t_1}}^q\right)^{\gamma_{t_1}}}}\right)\right) \otimes \left(\left(m_{t_2}^{\gamma_{t_2}}, u_{m_{t_2}}^{\gamma_{t_2}}\right), \left(\sqrt[q]{1 - \left(1 - n_{t_2}^q\right)^{\gamma_{t_2}}}, e^{i2\pi\sqrt[q]{1-\left(1-u_{n_{t_2}}^q\right)^{\gamma_{t_2}}}}\right)\right)$$

$$Cq{-}ROFDWG(\sigma_{t_1}, \sigma_{t_2}) = \left(\left(\prod_{k=1}^{2} m_{t_k}^{\gamma_{t_k}}, e^{\prod_{k=1}^{2} u_{m_{t_k}}^{\gamma_{t_k}}}\right), \left(\sqrt[q]{1 - \prod_{k=1}^{2}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{2}\left(1-u_{n_{t_k}}^q\right)^{\gamma_{t_k}}}}\right)\right).$$

Hence the assertion holds true for s=2.
Next we suppose the theory is valid $s = n > 2$ then:

$$Cq - ROFDWG(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_n}) = \left(\left(\prod_{k=1}^{n} m_{t_k}^{\gamma_{t_k}}, e^{\prod_{k=1}^{n} u_{m_{t_k}}^{\gamma_{t_k}}}\right), \left(\sqrt[q]{1 - \prod_{k=1}^{n}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{n}\left(1-u_{n_{t_k}}^q\right)^{\gamma_{t_k}}}}\right)\right).$$

Next, if s=n+1, then

$$Cq - ROFDWG\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_n}, \sigma_{t_{n+1}}\right) = \sigma_{t_1}^{\gamma_{t_1}} \otimes \sigma_{t_2}^{\gamma_{t_2}} \otimes ... \otimes \sigma_{t_n}^{\gamma_{t_n}} \otimes \sigma_{t_{n+1}}^{\gamma_{t_{n+1}}}$$

$$= \left(\left(\prod_{k=1}^{n+1} m_{t_k}^{\gamma_{t_k}}, e^{\prod_{k=1}^{n+1} u_{m_{t_k}}^{\gamma_{t_k}}}\right), \left(\sqrt[q]{1 - \prod_{k=1}^{n+1}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{n+1}\left(1-u_{n_{t_k}}^q\right)^{\gamma_{t_k}}}}\right)\right).$$

This illustrates that the theorem holds for s=n+1. As a result, we can conclude that statement true for for all positive integers s. The subsequent theorem investigates the idempotent characteristic of the Cq-ROFDWG operator. Consider a set of Cq-ROFNs $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$ for k=1,2,...,s. If $\sigma_{t_k} = \sigma_{t_l}$ for all k and some $l \in \{1, 2, ..., s\}$, where $\sigma_{t_l} = \left(\left(m_{t_l}, e^{i2\pi u_{m_{t_l}}}\right), \left(n_{t_l}, e^{i2\pi u_{n_{t_l}}}\right)\right)$ and $\gamma_{t_k} = [\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s}]^T$ represents the weight vector associated with time periods t_k, such that $\gamma_{t_k} \in [0, 1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$, then

$$Cq - ROFDWG\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) = \sigma_{t_l}.$$

Proof can be obtained by using the same approach employed in Theorem 2. The subsequent theorem investigates the boundedness characteristic of the Cq-ROFDWG operator.
Let $\sigma_t^- = \left(\left(min\left\{m_{t_k}\right\}, min\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right), \left(max\left\{n_{t_k}\right\}, max\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)\right)$
and $\sigma_t^+ = \left(\left(max\left\{m_{t_k}\right\}, max\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right), \left(min\left\{n_{t_k}\right\}, min\left\{e^{i2\pi u_{n_{t_k}}}\right\}\right)\right)$ be the lower and upper bound of the Cq-ROFNs $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i2\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i2\pi u_{n_{t_k}}}\right)\right)$, where k takes on values form 1 to s. Let $\gamma_{t_k} = (\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s})^T$ represents the weight vector associated with time periods t_k, such that $\gamma_{t_k} \in [0, 1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$. Then,

$$\sigma_t^- \leq Cq - ROFDWG\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) \leq \sigma_t^+.$$

Let us apply Cq-ROFDWG operator on Cq-ROFNs σ_{t_k} for all k.

$$Cq - ROFDWG\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) = \left(\left(m_t, e^{i2\pi u_{m_t}}\right), \left(n_t, e^{i\pi u_{n_t}}\right)\right)$$

For each m_{t_k}

$$min\left\{m_{t_k}\right\} \leq m_{t_k} \leq max\left\{m_{t_k}\right\}$$

$$\implies \prod_{k=1}^{s}\left(min\left\{m_{t_k}\right\}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(m_{t_k}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(max\left\{m_{t_k}\right\}\right)^{\gamma_{t_k}}$$

$$\implies \left(min\left\{m_{t_k}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(m_{t_k}\right)^{\gamma_{t_k}} \leq \left(max\left\{m_{t_k}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$\implies min\left\{m_{t_k}\right\} \leq m_t \leq max\left\{m_{t_k}\right\}$$

For each

$$u_{m_{t_k}}$$

$$min\left\{e^{i2\pi u_{m_{t_k}}}\right\} \leq e^{i2\pi u_{m_{t_k}}} \leq max\left\{e^{i2\pi u_{m_{t_k}}}\right\}$$

$$\implies \prod_{k=1}^{s}\left(min\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(e^{i2\pi u_{m_{t_k}}}\right)^{\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(max\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right)^{\gamma_{t_k}}$$

$$min\left\{e^{i2\pi u_{m_{t_k}}}\right\} \leq e^{i2\pi u_{m_{t_k}}} \leq max\left\{e^{i2\pi u_{m_{t_k}}}\right\}$$

$$\implies \left(min\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \leq \prod_{k=1}^{s}\left(e^{i2\pi u_{m_{t_k}}}\right)^{\gamma_{t_k}} \leq \left(max\left\{e^{i2\pi u_{m_{t_k}}}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$\implies min\left\{e^{i2\pi u_{m_{t_k}}}\right\} \leq u_{m_t} \leq max\left\{e^{i2\pi u_{m_{t_k}}}\right\}$$

Moreover,

$$min\left\{n_{t_k}\right\} \leq n_{t_k} \leq max\left\{n_{t_k}\right\}$$

$$\implies min\left\{n_{t_k}^q\right\} \leq n_{t_k}^q \leq max\left\{n_{t_k}^q\right\}$$

$$\implies 1 - max\left\{n_{t_k}^q\right\} \geq 1 - n_{t_k}^q \geq 1 - min\left\{n_{t_k}^q\right\}$$

$$\implies \prod_{k=1}^{s}\left(1 - max\left\{n_{t_k}^q\right\}\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - min\left\{n_{t_k}^q\right\}\right)^{\gamma_{t_k}}$$

$$\implies \left(1 - max\left\{n_{t_k}^q\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}} \geq \left(1 - min\left\{n_{t_k}^q\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$\implies \left(1 - max\left\{n_{t_k}^q\right\}\right) \geq \prod_{k=1}^{s}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}} \geq \left(1 - min\left\{n_{t_k}^q\right\}\right)$$

$$\implies min\left\{n_{t_k}^q\right\} \leq 1 - \prod_{k=1}^{s}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}} \leq max\left\{n_{t_k}^q\right\}$$

$$\implies \sqrt[q]{min\left\{n_{t_k}^q\right\}} \leq \sqrt[q]{1 - \prod_{k=1}^{s}\left(1 - n_{t_k}^q\right)^{\gamma_{t_k}}} \leq \sqrt[q]{max\left\{n_{t_k}^q\right\}}$$

Therefore,

$$min\left\{n_{t_k}\right\} \leq n_t \leq max\left\{n_{t_k}\right\}$$

Furthermore,

$$min\left\{u_{n_{t_k}}\right\} \leq u_{n_{t_k}} \leq max\left\{u_{n_{t_k}}\right\}$$

$$\implies min\left\{e^{i2\pi u_{n_{t_k}}^q}\right\} \leq e^{i2\pi u_{n_{t_k}}^q} \leq max\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}$$

$$\implies 1 - max\left\{e^{i2\pi u_{n_{t_k}}^q}\right\} \geq 1 - e^{i2\pi u_{n_{t_k}}^q} \geq 1 - min\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}$$

$$\implies \prod_{k=1}^{s}\left(1 - max\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - e^{i2\pi u_{n_{t_k}}^q}\right)^{\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - min\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}\right)^{\gamma_{t_k}}$$

$$\implies \left(1 - max\left\{e^{i2\pi u_{u_{t_k}}^q}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}} \geq \prod_{k=1}^{s}\left(1 - e^{i2\pi u_{n_{t_k}}^q}\right)^{\gamma_{t_k}} \geq \left(1 - min\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}\right)^{\sum_{k=1}^{s}\gamma_{t_k}}$$

$$\implies \left(1 - max\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}\right) \geq \prod_{k=1}^{s}\left(1 - e^{i2\pi u_{n_{t_k}}^q}\right)^{\gamma_{t_k}} \geq \left(1 - min\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}\right)$$

$$\implies min\left\{e^{i2\pi u_{n_{t_k}}^q}\right\} \leq 1 - \prod_{k=1}^{s}\left(1 - e^{i2\pi u_{n_{t_k}}^q}\right)^{\gamma_{t_k}} \leq max\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}$$

$$\implies \sqrt[q]{min\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}} \leq \sqrt[q]{1 - \prod_{k=1}^{s}\left(1 - e^{i2\pi u_{n_{t_k}}^q}\right)^{\gamma_{t_k}}} \leq \sqrt[q]{max\left\{e^{i2\pi u_{n_{t_k}}^q}\right\}}$$

Therefore,

$$min\left\{e^{i2\pi u_{n_{t_k}}}\right\} \leq e^{i2\pi u_{n_t}} \leq max\left\{e^{i2\pi u_{n_{t_k}}}\right\}$$

Hence by employing Definition 16, we obtain that

$$\sigma_t^- \leq Cq - ROFDWG\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) \leq \sigma_t^+.$$

The subsequent theorem investigates the monotonic characteristic of the Cq-ROFDWG operator. Consider sets of two Cq-ROFNs, denoted as $\sigma_{t_k} = \left(\left(m_{t_k}, e^{i\pi u_{m_{t_k}}}\right), \left(n_{t_k}, e^{i\pi u_{n_{t_k}}}\right)\right)$ and $\sigma'_{t_k} = \left(\left(m'_{t_k}, e^{i\pi u'_{m_{t_k}}}\right), \left(n'_{t_k}, e^{i2\pi u'_{n_{t_k}}}\right)\right)$ for k=1,2,...,s.Let $\gamma_{t_k} = [\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s}]^T$ represents the weight vector associated with time periods t_k, such that $\gamma_{t_k} \in [0,1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$. If for each t_k: $m_{t_k} \leq m'_{t_k}, e^{i2\pi u_{m_{t_k}}} \leq e^{i2\pi u'_{m_{t_k}}}$, and $n_{t_k} \geq n'_{t_k}, e^{i2\pi u_{n_{t_k}}} \geq e^{i2\pi u'_{n_{t_k}}}$. Then,

$$Cq - ROFDWA\left(\sigma_{t_1}, \sigma_{t_2}, ..., \sigma_{t_s}\right) \leq Cq - ROFDWA\left(\sigma'_{t_1}, \sigma'_{t_2}, ..., \sigma'_{t_s}\right).$$

Proof of this theorem is similar to Theorem 4.

Remarks: Linguistic and metaphysical interpretation of above theorems.

In Theorem 1 and 5, it is proved that when we apply DWA and DWG operators to a finite collection of Cq-ROFNs, their aggregated value give a Cq-ROFN. Theorem 2 to 6 verify the idempotency property, this property ensures that applying the weighted aggregation operator to the same input Cq-ROFS multiple time produces the same result as applying it once. This consistency and stability are essential in decision making process, providing predictability and reliability in the aggregated outcome. Theorem 3 and 7 satisfy the monotonicity property, this property ensures the output of the aggregation operation behaves consistently with changes in the input fuzzy sets membership and non-membership values. This property also guarantees the preservation of order in the aggregation process. If the input Cq-ROFS exhibits certain order in terms of their membership and non-membership values, the monotonicity property ensures that the order is maintained in the aggregated output. The boundedness property is established in Theorem 4 and 8. This property guarantees that the results generated by the weighted aggregation operator are contained within specific limits. It is important to ensure that the aggregation process produces meaningful and precisely defined outcomes.

4. Algorithm of newly defined complex q-rung orthopair fuzzy dynamic aggregation operators in multi attribute decision making problem

In this section, we develop a method based on proposed aggregation operators to deal with MADM issue.

- Let $\theta_i = \{\theta_1, \theta_2, ..., \theta_m\}$ be a discrete collection of alternatives.
- Suppose that $\delta_j = \{\delta_1, \delta_2, ..., \delta_n\}$ denotes the set of attributes and $w = [w_1, w_2, ..., w_n]^T$ is the associated weighted vector, where $w_j \in [0,1]$ such that $\sum_{j=1}^{n} w_k = 1$.
- Let t_k, where k=1,2,...,s, denote s number of time periods with weight vector $\gamma_{t_k} = [\gamma_{t_1,\gamma_{t_2}}, ..., \gamma_{t_s}]^T$ such that $\gamma_{t_k} \in [0,1]$ and $\sum_{k=1}^{s} \gamma_{t_k} = 1$.
- Suppose that $Z_{t_k} = [z_{ij(t_k)}]_{m\times n} = \left(\left(m_{ij(t_k)}, e^{i2\pi u_{m_{ij(t_k)}}}\right), \left(n_{ij(t_k)}, e^{i2\pi u_{n_{ij(t_k)}}}\right)\right)_{m\times n}$ is the Cq-ROF decision matrix at t_k. Here $\left(m_{ij(t_k)}, e^{i2\pi u_{m_{ij(t_k)}}}\right)$ shows how much alternative θ_i meets the

requirement for attribute δ_j during time interval t_k and $\left(n_{ij(t_k)}, e^{i2\pi u_{n_{ij(t_k)}}}\right)$ shows how much alternative θ_i doesn't meet the requirement for attribute δ_j during time interval t_k. Additionally, $m_{ij(t_k)}, u_{m_{ij(t_k)}}, n_{ij(t_k)}, u_{n_{ij(t_k)}} \in [0,1]$ such that $m^q_{ij(t_k)} + n^q_{ij(t_k)} \le 1$, and $u^q_{m_{ij(t_k)}} + u^q_{{ij(t_k)}} \le 1$, where i varies from 1 to m and j varies from 1 to n.

By utilizing the aforementioned decision data, we devise an operational approach to rank and choose the most favorable choice.

A. Procedure for Cq-ROFDWA

Step 1: Apply Cq-ROFDWA operator on the matrix

$$Z_{t_k} = z_{ij(t_k)} = \left(\left(m_{ij(t_k)}, e^{i2\pi u_{m_{ij(t_k)}}}\right), \left(n_{ij(t_k)}, e^{i2\pi u_{n_{ij(t_k)}}}\right)\right)$$
$$= Cq - ROFDWA\left(z_{ij(t_1)}, z_{ij(t_2)}, ..., z_{ij(t_s)}\right)$$
$$= \oplus^s_{k=1}\gamma_{t_k}F_{t_k}.$$

Apply Theorem 1 to above relationship:

$$z_{ij(t_k)} = \left(\left(\sqrt[q]{1-\prod_{k=1}^{s}\left(1-m^q_{t_k}\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u^q_{m_{t_k}}\right)^{\gamma_{t_k}}}}\right), \left(\prod_{k=1}^{s} n^{\gamma_{t_k}}_{t_k}, e^{\prod_{k=1}^{s} u^{\gamma_{t_k}}_{n_{t_k}}}\right)\right).$$

This aggregation combines all the Cq-ROF decision matrices Z_{t_k}, as a collective decision matrix Z, where$\gamma_{t_k} = [\gamma_{t_1}, \gamma_{t_2}, ..., \gamma_{t_s}]^T$ denotes the associated weight vector for the time period t_k.

Step 2: Apply Cq-ROFWA operators on collective Cq-ROF information given in matrix Z as follows:

$$Z = z_i = \left(\left(m_i, e^{i2\pi u_{m_i}}\right), \left(n_i, e^{i2\pi u_{n_i}}\right)\right) = Cq - ROFWA\left(z_{i1}, z_{i2}, ..., z_{in}\right)$$

$$= \left(\left(\left(1-\prod_{j=1}^{n}\left(1-m^q_i\right)^{w_j}\right)^{\frac{1}{q}} . e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-u^q_{m_i}\right)^{w_j}\right)^{\frac{1}{q}}}\right), \prod_{j=1}^{n} n^{w_k}_i . e^{i2\pi\left(\prod_{j=1}^{n} u^{w_j}_{n_i}\right)}\right).$$

This action derive the collective overall preference value z_i for the alternativeθ_i, where $w = (w_1, w_2, ..., w_n)^T$ and $w_j \in [0,1]$ are the weight vectors for the attributes. **Step 3:** Use Definition to calculate score for each attribute.

Step 4: Rank all the alternatives and identify the optimal one.

B.Procedure for Cq-ROFDWG

Step 1. Apply Cq-ROFDWG operator on the matrix Z_{t_k}:

$$Z_{t_k} = z_{ij(t_k)} = \left(\left(m_{ij(t_k)}, e^{i2\pi u_{m_{ij(t_k)}}}\right), \left(n_{ij(t_k)}, e^{i2\pi u_{n_{ij(t_k)}}}\right)\right)$$
$$= Cq - ROFDWA\left(z_{ij(t_1)}, z_{ij(t_2)}, ..., z_{ij(t_s)}\right)$$
$$= \oplus^s_{k=1}\gamma_{t_k}F_{t_k}.$$

Apply Theorem 1 to above relationship:

$$= \left(\left(\prod_{k=1}^{s} m^{\gamma_{t_k}}_{t_k}, e^{\prod_{k=1}^{s} u^{\gamma_{t_k}}_{m_{t_k}}}\right), \left(\sqrt[q]{1-\prod_{k=1}^{s}\left(1-n^q_{t_k}\right)^{\gamma_{t_k}}}, e^{\sqrt[q]{1-\prod_{k=1}^{s}\left(1-u^q_{t_k}\right)^{\gamma_{t_k}}}}\right)\right).$$

This aggregation combines all the Cq-ROF decision matrices Z_{t_k}, as a collective decision matrix Z.
Step 2: Apply Cq-ROFWG operators on collective Cq-ROF information given in matrix Z as follows:

$$Z = z_i = \left(\left(m_i, e^{i2\pi u_{m_i}}\right), \left(n_i, e^{i2\pi u_{n_i}}\right)\right) = Cq - ROFDWG\left(z_{i1}, z_{i2}, ..., z_{in}\right)$$

$$= \left(\prod_{j=1}^{n} m_i^{w_j} . e^{i2\pi\left(\prod_{j=1}^{n} u_{m_i}^{w_j}\right)^{\frac{1}{q}}}, \left(1 - \prod_{j=1}^{n}\left(1 - n_i^q\right)^{w_j}\right)^{\frac{1}{q}} . e^{i2\pi\left(1 - \prod_{j=1}^{n}\left(1 - u_{n_i}^q\right)^{w_j}\right)^{\frac{1}{q}}}\right).$$

This action derive the collective overall preference valuez_i for the alternativeθ_i, where $w = (w_1, w_2, ..., w_n)^T$ and $w_j \in [0, 1]$ are the weight vectors for the attributes.
Step 3: Use Definition to calculate score for each attribute.
Step 4: Rank all the alternatives and identify the optimal one.

5. Illustrative Example

When a rescue department is contemplating an upgrade to their communication system to bolster coordination and effectiveness during emergencies, they undertake a comprehensive and intricate decision-making journey. This process begins with a careful assessment of their current communication infrastructure, which involves a thorough examination of existing equipment, technologies, and protocols. They scrutinize how well these components support their operations in real-life emergency scenarios, identifying any limitations or gaps that may hinder their response capabilities. This initial evaluation is often informed by feedback from personnel who regularly use these systems, offering insights into the practical challenges and inefficiencies they encounter on the ground. As they move forward, the department engages in a deep dive into the latest advancements in communication technology. This exploration includes researching a wide range of options, from the most recent advancements in radio and satellite communication to state-of-the-art mobile data terminals and software solutions for incident management. They explore the potential benefits and drawbacks of emerging technologies, seeking innovations that could enhance real-time information sharing, improve signal reliability, and integrate seamlessly with existing systems. This stage also involves consultations with technology vendors and experts, attending industry conferences, and reviewing case studies from other rescue departments that have successfully implemented new systems. Following are the key criterias in this regard, these criteria help ensure that the upgraded communication system effectively meets the department's needs for emergency response and management.
δ_1=**Technological Advancement:** Modernity and capability of the technology.
δ_2=**Reliability:** Consistency and dependability of the system under various conditions.
δ_3=**Coverage and Range:** Geographic and operational area coverage.
δ_4=**Ease of Use:** User-friendliness and intuitiveness of the system.
δ_5=**Cost:** Initial investment, maintenance, and long-term operational expenses.
When considering alternatives for upgrading a rescue department's communication system, there are various technologies and approaches that can be explored. Here are several alternatives, each addressing different aspects of communication needs:
θ_1=**Land Mobile Radio Systems (LMR):** A standardized digital radio system used for public safety communications. Offers secure, reliable, and interoperable communication.
θ_2=**Satellite Communication Systems:** Provide reliable communication in remote areas where other systems might not reach. Useful for high-mobility scenarios but can be expensive.
θ_3=**Mesh Network Systems:** Create self-healing, decentralized networks that can expand dynamically. Useful for establishing communication in disaster zones or large areas with limited infrastructure.

θ_4=**Emergency Management Software:** Software for real-time tracking and visualization of incidents and resources.
θ_5=**Public Safety Answering Point (PSAP) Upgrades:** Upgrading or integrating advanced systems for better incident management and resource allocation.
These alternatives provide a range of solutions from traditional radio systems to cutting-edge technology, allowing departments to choose the best fit for their specific operational needs and constraints. The evaluation of the five potential alternative can be conducted by utilizing the Cq-ROF data offered by the decision maker for the five attributes during the specified time intervals t_1, t_2 and t_3 presented in table 1, 2 and 3. Define the weight vector $\gamma_{t_k} = [0.25, 0.45, 0.30]$ corresponding to the time interval t_k where k=1,2,3 and the weight vector $w_j = [0.22, 0.15, 0.28, 0.16, 0.19]$ relating to the attributes δ_j where j=1, 2, 3, 4, 5. To choose the optimal communication system upgrade for a rescue department, we follow a structured algorithm that incorporates the identified criteria and evaluates various alternatives. This methodical approach ensures that we choose a solution that not only meets immediate requirements but also supports long-term operational goals.

Table 1
Decision matrix Z_{t_1}

Alternatives / Attributes	θ_1	θ_2	θ_3	θ_4	θ_5
δ_1	$\left((0.5, e^{i2\pi 0.6}),\ (0.6, e^{i2\pi 0.3})\right)$	$\left((0.7, e^{i2\pi 0.4}),\ (0.5, e^{i2\pi 0.2})\right)$	$\left((0.3, e^{i2\pi 0.8}),\ (0.7, e^{i2\pi 0.1})\right)$	$\left((0.5, e^{i2\pi 0.6}),\ (0.5, e^{i2\pi 0.3})\right)$	$\left((0.2, e^{i2\pi 0.7}),\ (0.4, e^{i2\pi 0.3})\right)$
δ_2	$\left((0.6, e^{i2\pi 0.6}),\ (0.2, e^{i2\pi 0.2})\right)$	$\left((0.2, e^{i2\pi 0.4}),\ (0.5, e^{i2\pi 0.7})\right)$	$\left((0.6, e^{i2\pi 0.5}),\ (0.1, e^{i2\pi 0.7})\right)$	$\left((0.4, e^{i2\pi 0.6}),\ (0.5, e^{i2\pi 0.4})\right)$	$\left((0.2, e^{i2\pi 0.6}),\ (0.3, e^{i2\pi 0.1})\right)$
δ_3	$\left((0.4, e^{i2\pi 0.5}),\ (0.2, e^{i2\pi 0.6})\right)$	$\left((0.3, e^{i2\pi 0.5}),\ (0.2, e^{i2\pi 0.4})\right)$	$\left((0.2, e^{i2\pi 0.6}),\ (0.7, e^{i2\pi 0.5})\right)$	$\left((0.4, e^{i2\pi 0.4}),\ (0.5, e^{i2\pi 0.4})\right)$	$\left((0.3, e^{i2\pi 0.6}),\ (0.4, e^{i2\pi 0.5})\right)$
δ_4	$\left((0.1, e^{i2\pi 0.7}),\ (0.5, e^{i2\pi 0.1})\right)$	$\left((0.6, e^{i2\pi 0.3}),\ (0.4, e^{i2\pi 0.4})\right)$	$\left((0.8, e^{i2\pi 0.5}),\ (0.3, e^{i2\pi 0.3})\right)$	$\left((0.4, e^{i2\pi 0.7}),\ (0.5, e^{i2\pi 0.2})\right)$	$\left((0.3, e^{i2\pi 0.5}),\ (0.6, e^{i2\pi 0.3})\right)$
δ_5	$\left((0.4, e^{i2\pi 0.4}),\ (0.5, e^{i2\pi 0.6})\right)$	$\left((0.3, e^{i2\pi 0.6}),\ (0.7, e^{i2\pi 0.5})\right)$	$\left((0.8, e^{i2\pi 0.4}),\ (0.2, e^{i2\pi 0.8})\right)$	$\left((0.6, e^{i2\pi 0.4}),\ (0.3, e^{i2\pi 0.4})\right)$	$\left((0.1, e^{i2\pi 0.4}),\ (0.8, e^{i2\pi 0.5})\right)$

Table 2
Decision matrix Z_{t_2}

Alternatives	Attributes	Values
δ_1	θ_1	$\left(\left(0.4, e^{i2\pi 0.3}\right), \left(0.6, e^{i2\pi 0.5}\right)\right)$
δ_1	θ_2	$\left(\left(0.5, e^{i2\pi 0.3}\right), \left(0.4, e^{i2\pi 0.7}\right)\right)$
δ_1	θ_3	$\left(\left(0.9, e^{i2\pi 0.6}\right), \left(0.2, e^{i2\pi 0.5}\right)\right)$
δ_1	θ_4	$\left(\left(0.4, e^{i2\pi 0.6}\right), \left(0.5, e^{i2\pi 0.3}\right)\right)$
δ_1	θ_5	$\left(\left(0.6, e^{i2\pi 0.2}\right), \left(0.5, e^{i2\pi 0.6}\right)\right)$
δ_2	θ_1	$\left(\left(0.3, e^{i2\pi 0.7}\right), \left(0.6, e^{i2\pi 0.8}\right)\right)$
δ_2	θ_2	$\left(\left(0.1, e^{i2\pi 0.7}\right), \left(0.6, e^{i2\pi 0.4}\right)\right)$
δ_2	θ_3	$\left(\left(0.2, e^{i2\pi 0.3}\right), \left(0.9, e^{i2\pi 0.1}\right)\right)$
δ_2	θ_4	$\left(\left(0.5, e^{i2\pi 0.6}\right), \left(0.4, e^{i2\pi 0.6}\right)\right)$
δ_2	θ_5	$\left(\left(0.8, e^{i2\pi 0.2}\right), \left(0.7, e^{i2\pi 0.6}\right)\right)$
δ_3	θ_1	$\left(\left(0.6, e^{i2\pi 0.2}\right), \left(0.5, e^{i2\pi 0.5}\right)\right)$
δ_3	θ_2	$\left(\left(0.6, e^{i2\pi 0.2}\right), \left(0.8, e^{i2\pi 0.2}\right)\right)$
δ_3	θ_3	$\left(\left(0.7, e^{i2\pi 0.3}\right), \left(0.4, e^{i2\pi 0.9}\right)\right)$
δ_3	θ_4	$\left(\left(0.2, e^{i2\pi 0.5}\right), \left(0.7, e^{i2\pi 0.3}\right)\right)$
δ_3	θ_5	$\left(\left(0.6, e^{i2\pi 0.4}\right), \left(0.4, e^{i2\pi 0.7}\right)\right)$
δ_4	θ_1	$\left(\left(0.4, e^{i2\pi 0.5}\right), \left(0.8, e^{i2\pi 0.1}\right)\right)$
δ_4	θ_2	$\left(\left(0.4, e^{i2\pi 0.5}\right), \left(0.7, e^{i2\pi 0.3}\right)\right)$
δ_4	θ_3	$\left(\left(0.2, e^{i2\pi 0.7}\right), \left(0.4, e^{i2\pi 0.3}\right)\right)$
δ_4	θ_4	$\left(\left(0.5, e^{i2\pi 0.8}\right), \left(0.3, e^{i2\pi 0.6}\right)\right)$
δ_4	θ_5	$\left(\left(0.4, e^{i2\pi 0.4}\right), \left(0.5, e^{i2\pi 0.2}\right)\right)$
δ_5	θ_1	$\left(\left(0.4, e^{i2\pi 0.7}\right), \left(0.2, e^{i2\pi 0.7}\right)\right)$
δ_5	θ_2	$\left(\left(0.1, e^{i2\pi 0.8}\right), \left(0.3, e^{i2\pi 0.2}\right)\right)$
δ_5	θ_3	$\left(\left(0.5, e^{i2\pi 0.5}\right), \left(0.4, e^{i2\pi 0.6}\right)\right)$
δ_5	θ_4	$\left(\left(0.2, e^{i2\pi 0.9}\right), \left(0.1, e^{i2\pi 0.8}\right)\right)$
δ_5	θ_5	$\left(\left(0.4, e^{i2\pi 0.5}\right), \left(0.6, e^{i2\pi 0.2}\right)\right)$

Table 3
Decision matrix Z_{t_3}

Alternatives / Attributes	θ_1	θ_2	θ_3	θ_4	θ_5
δ_1	$\left(\left(0.2, e^{i2\pi 0.6}\right), \left(0.4, e^{i2\pi 0.5}\right)\right)$	$\left(\left(0.6, e^{i2\pi 0.3}\right), \left(0.5, e^{i2\pi 0.4}\right)\right)$	$\left(\left(0.1, e^{i2\pi 0.5}\right), \left(0.4, e^{i2\pi 0.5}\right)\right)$	$\left(\left(0.2, e^{i2\pi 0.6}\right), \left(0.5, e^{i2\pi 0.3}\right)\right)$	$\left(\left(0.7, e^{i2\pi 0.4}\right), \left(0.1, e^{i2\pi 0.4}\right)\right)$
δ_2	$\left(\left(0.5, e^{i2\pi 0.2}\right), \left(0.4, e^{i2\pi 0.5}\right)\right)$	$\left(\left(0.6, e^{i2\pi 0.3}\right), \left(0.5, e^{i2\pi 0.6}\right)\right)$	$\left(\left(0.3, e^{i2\pi 0.7}\right), \left(0.2, e^{i2\pi 0.1}\right)\right)$	$\left(\left(0.4, e^{i2\pi 0.6}\right), \left(0.5, e^{i2\pi 0.4}\right)\right)$	$\left(\left(0.6, e^{i2\pi 0.3}\right), \left(0.4, e^{i2\pi 0.5}\right)\right)$
δ_3	$\left(\left(0.6, e^{i2\pi 0.4}\right), \left(0.3, e^{i2\pi 0.5}\right)\right)$	$\left(\left(0.2, e^{i2\pi 0.4}\right), \left(0.7, e^{i2\pi 0.6}\right)\right)$	$\left(\left(0.4, e^{i2\pi 0.3}\right), \left(0.2, e^{i2\pi 0.7}\right)\right)$	$\left(\left(0.5, e^{i2\pi 0.5}\right), \left(0.6, e^{i2\pi 0.4}\right)\right)$	$\left(\left(0.3, e^{i2\pi 0.2}\right), \left(0.6, e^{i2\pi 0.7}\right)\right)$
δ_4	$\left(\left(0.1, e^{i2\pi 0.6}\right), \left(0.6, e^{i2\pi 0.2}\right)\right)$	$\left(\left(0.3, e^{i2\pi 0.7}\right), \left(0.2, e^{i2\pi 0.4}\right)\right)$	$\left(\left(0.9, e^{i2\pi 0.7}\right), \left(0.5, e^{i2\pi 0.2}\right)\right)$	$\left(\left(0.6, e^{i2\pi 0.4}\right), \left(0.4, e^{i2\pi 0.7}\right)\right)$	$\left(\left(0.8, e^{i2\pi 0.3}\right), \left(0.3, e^{i2\pi 0.5}\right)\right)$
δ_5	$\left(\left(0.2, e^{i2\pi 0.7}\right), \left(0.4, e^{i2\pi 0.6}\right)\right)$	$\left(\left(0.4, e^{i2\pi 0.3}\right), \left(0.1, e^{i2\pi 0.9}\right)\right)$	$\left(\left(0.4, e^{i2\pi 0.7}\right), \left(0.3, e^{i2\pi 0.6}\right)\right)$	$\left(\left(0.6, e^{i2\pi 0.5}\right), \left(0.8, e^{i2\pi 0.5}\right)\right)$	$\left(\left(0.3, e^{i2\pi 0.6}\right), \left(0.4, e^{i2\pi 0.1}\right)\right)$

Step 1: Apply the Cq-ROFDWA operator to consolidate all the Cq-ROF decision matrices Z_{t_k} into a unified Cq-ROF decision matrix Z displayed in Table 4.

Table 4
Decision matrix Z using Cq-ROFDWA

Alternatives / Attributes	θ_1	θ_2	θ_3	θ_4	θ_5
δ_1	$\left(\left(0.3987, e^{i2\pi 0.5141}\right), \left(0.5313, e^{i2\pi 0.4401}\right)\right)$	$\left(\left(0.5962, e^{i2\pi 0.3314}\right), \left(0.4523, e^{i2\pi 0.4326}\right)\right)$	$\left(\left(0.7653, e^{i2\pi 0.6544}\right), \left(0.3369, e^{i2\pi 0.3343}\right)\right)$	$\left(\left(0.3987, e^{i2\pi 0.6000}\right), \left(0.5000, e^{i2\pi 0.3000}\right)\right)$	$\left(\left(0.5957, e^{i2\pi 0.4939}\right), \left(0.2918, e^{i2\pi 0.4468}\right)\right)$
δ_2	$\left(\left(0.4747, e^{i2\pi 0.6063}\right), \left(0.4036, e^{i2\pi 0.4913}\right)\right)$	$\left(\left(0.4174, e^{i2\pi 0.5774}\right), \left(0.5428, e^{i2\pi 0.5195}\right)\right)$	$\left(\left(0.4123, e^{i2\pi 0.5403}\right), \left(0.3309, e^{i2\pi 0.1626}\right)\right)$	$\left(\left(0.4513, e^{i2\pi 0.6000}\right), \left(0.4523, e^{i2\pi 0.4801}\right)\right)$	$\left(\left(0.6898, e^{i2\pi 0.4123}\right), \left(0.4788, e^{i2\pi 0.3629}\right)\right)$
δ_3	$\left(\left(0.5651, e^{i2\pi 0.3808}\right), \left(0.3411, e^{i2\pi 0.5233}\right)\right)$	$\left(\left(0.4819, e^{i2\pi 0.3803}\right), \left(0.5434, e^{i2\pi 0.3307}\right)\right)$	$\left(\left(0.5750, e^{i2\pi 0.4274}\right), \left(0.3737, e^{i2\pi 0.7206}\right)\right)$	$\left(\left(0.4880, e^{i2\pi 0.4665}\right), \left(0.6144, e^{i2\pi 0.3514}\right)\right)$	$\left(\left(0.4892, e^{i2\pi 0.4461}\right), \left(0.4518, e^{i2\pi 0.6435}\right)\right)$
δ_4	$\left(\left(0.3104, e^{i2\pi 0.5962}\right), \left(0.6525, e^{i2\pi 0.1231}\right)\right)$	$\left(\left(0.4548, e^{i2\pi 0.5598}\right), \left(0.4180, e^{i2\pi 0.3514}\right)\right)$	$\left(\left(0.7589, e^{i2\pi 0.6651}\right), \left(0.3980, e^{i2\pi 0.2656}\right)\right)$	$\left(\left(0.5179, e^{i2\pi 0.7120}\right), \left(0.3716, e^{i2\pi 0.4774}\right)\right)$	$\left(\left(0.6060, e^{i2\pi 0.4098}\right), \left(0.4489, e^{i2\pi 0.2914}\right)\right)$
δ_5	$\left(\left(0.3622, e^{i2\pi 0.6559}\right), \left(0.3097, e^{i2\pi 0.6431}\right)\right)$	$\left(\left(0.2993, e^{i2\pi 0.6870}\right), \left(0.2667, e^{i2\pi 0.3949}\right)\right)$	$\left(\left(0.6112, e^{i2\pi 0.5682}\right), \left(0.3085, e^{i2\pi 0.6447}\right)\right)$	$\left(\left(0.5045, e^{i2\pi 0.7802}\right), \left(0.2456, e^{i2\pi 0.5843}\right)\right)$	$\left(\left(0.3350, e^{i2\pi 0.5179}\right), \left(0.5709, e^{i2\pi 0.2043}\right)\right)$

Step 2: Apply the Cq-ROFWA operator on the values of the matrix Z to obtain the overall values of all the alternates.
$\theta_1 = ((0.6200, e^{i2\pi 0.5600}), (0.4034, e^{i2\pi 0.3834}))$, $\theta_2 = ((0.5158, e^{i2\pi 0.5559}), (0.4198, e^{i2\pi 0.3419}))$,
$\theta_3 = ((0.5223, e^{i2\pi 0.4228}), (0.4349, e^{i2\pi 0.5214}))$, $\theta_4 = ((0.6077, e^{i2\pi 0.6145}), (0.4523, e^{i2\pi 0.2614}))$,
$\theta_5 = ((0.4793, e^{i2\pi 0.6493}), (0.3275, e^{i2\pi 0.4736}))$.
Step 3: Determine the ranking of all the alternatives by calculating the score values of the Cq-ROF values: $S_1(\theta_1) = 0.1460$, $S_2(\theta_2) = 0.09752$, $S_3(\theta_3) = -0.00294$, $S_4(\theta_4) = 0.1730$, $S_5(\theta_5) = 0.1212$.

Step 4: Since $S_4(\theta_4) > S_1(\theta_1) > S_5(\theta_5) > S_2(\theta_2) > S_3(\theta_3)$, as a result the following is the alternative ranking order: $\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$. Hence, LMR is the best option.

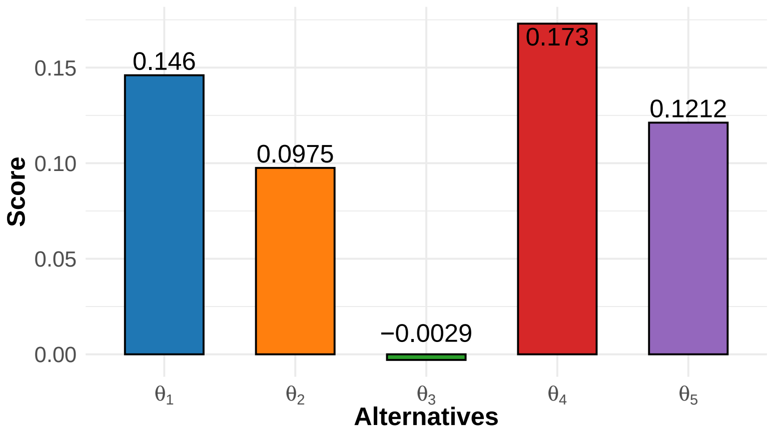

Fig. 1. Ranking Based on Scores Values by using Cq-ROFDWA

In a similar way, the above mentioned MADM problem is resolved as follows within the context of Cq-ROFDWG operator.

Step 1: Apply the Cq-ROFDWG operator to consolidate all the Cq-ROF decision matrices Z_{t_k} into a unified Cq-ROF decision matrix Z displayed in table 5.

Table 5
Decision matrix Z using Cq-ROFDWG

Alternatives / Attributes	θ_1	θ_2	θ_3	θ_4	θ_5
δ_1	$((0.3435, e^{i2\pi 0.4392}), (0.5573, e^{i2\pi 0.4665}))$	$((0.5744, e^{i2\pi 0.3223}), (0.4610, e^{i2\pi 0.5750}))$	$((0.3537, e^{i2\pi 0.6104}), (0.4939, e^{i2\pi 0.4572}))$	$((0.3435, e^{i2\pi 0.5999}), (0.5000, e^{i2\pi 0.2996}))$	$((0.4774, e^{i2\pi 0.3369}), (0.4198, e^{i2\pi 0.5029}))$
δ_2	$((0.4159, e^{i2\pi 0.4625}), (0.4973, e^{i2\pi 0.6774}))$	$((0.2036, e^{i2\pi 0.4720}), (0.5509, e^{i2\pi 0.5725}))$	$((0.2973, e^{i2\pi 0.4395}), (0.7639, e^{i2\pi 0.4649}))$	$((0.4423, e^{i2\pi 0.5999}), (0.4610, e^{i2\pi 0.5139}))$	$((0.5189, e^{i2\pi 0.2973}), (0.2578, e^{i2\pi 0.5183}))$
δ_3	$((0.5421, e^{i2\pi 0.3097}), (0.4080, e^{i2\pi 0.5298}))$	$((0.3629, e^{i2\pi 0.3097}), (0.7133, e^{i2\pi 0.4463}))$	$((0.4326, e^{i2\pi 0.3568}), (0.5042, e^{i2\pi 0.8073}))$	$((0.3131, e^{i2\pi 0.4401}), (0.6348, e^{i2\pi 0.3619}))$	$((0.4098, e^{i2\pi 0.3595}), (0.4826, e^{i2\pi 0.6651}))$
δ_4	$((0.8166, e^{i2\pi 0.5744}), (0.7041, e^{i2\pi 0.1474}))$	$((0.4060, e^{i2\pi 0.4868}), (0.5727, e^{i2\pi 0.3619}))$	$((0.4441, e^{i2\pi 0.6435}), (0.4195, e^{i2\pi 0.2772}))$	$((0.4995, e^{i2\pi 0.6285}), (0.5992, e^{i2\pi 0.5957}))$	$((0.4582, e^{i2\pi 0.3880}), (0.4949, e^{i2\pi 0.3667}))$
δ_5	$((0.3249, e^{i2\pi 0.6086}), (0.3808, e^{i2\pi 0.6509}))$	$((0.1995, e^{i2\pi 0.5547}), (0.4806, e^{i2\pi 0.7038}))$	$((0.5259, e^{i2\pi 0.5231}), (0.3394, e^{i2\pi 0.6721}))$	$((0.3660, e^{i2\pi 0.6161}), (0.5843, e^{i2\pi 0.6810}))$	$((0.2594, e^{i2\pi 0.4995}), (0.6428, e^{i2\pi 0.3320}))$

Step 2: Apply the Cq-ROFDWG operator on the values of the matrix Z to obtain the overall values of all the alternates.
$\theta_1 = ((0.2355, e^{i2\pi 0.5966}), (0.5518, e^{i2\pi 0.5743}))$, $\theta_2 = ((0.2249, e^{i2\pi 0.5701}), (0.6073, e^{i2\pi 0.6372}))$,
$\theta_3 = ((0.2209, e^{i2\pi 0.5535}), (0.6227, e^{i2\pi 0.6766}))$, $\theta_4 = ((0.2355, e^{i2\pi 0.6527}), (0.6388, e^{i2\pi 0.4531}))$,
$\theta_5 = ((0.1295, e^{i2\pi 0.6773}), (0.6685, e^{i2\pi 0.6555}))$.

Step 3: Determine the ranking of all the alternatives by calculating the score values of the Cq-ROF values. $S_1(\theta_1) = -0.06597$, $S_2(\theta_2) = -0.1430$, $S_3(\theta_3) = -0.1854$, $S_4(\theta_4) = -0.03134$, $S_5(\theta_5) = -0.1337$.

Step 4: Since $S_4(\theta_4) > S_1(\theta_1) > S_5(\theta_5) > S_2(\theta_2) > S_3(\theta_3)$, as a result the following is the alternative ranking order: $\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$.

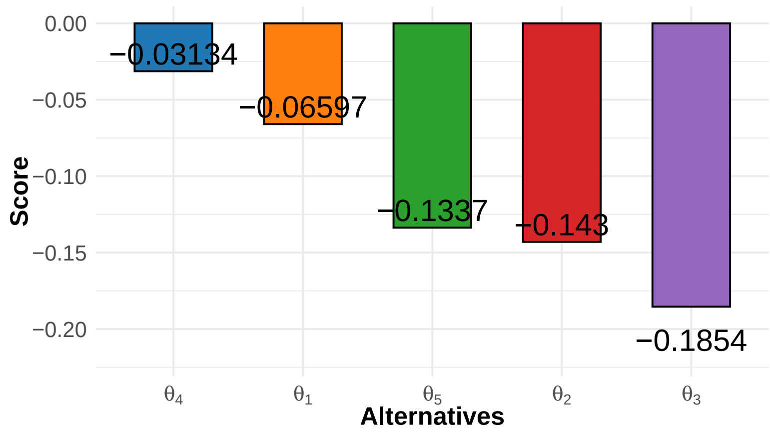

Fig. 2. Ranking Based on Scores Values by using Cq-ROFDWG

6. Influence of the parameters

This section focuses on conducting a sensitivity analysis to assess how different parameters affect the ranking outcomes. Table 6 and 7 showing the results of ranking using different values for q.

Table 6
Results with different values of q for the Cq-ROFDWA operator

Value of q	Ranking
$q = 5$	$\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$
$q = 12$	$\theta_1 > \theta_4 > \theta_5 > \theta_2 > \theta_3$
$q = 18$	$\theta_1 > \theta_4 > \theta_5 > \theta_2 > \theta_3$

Table 7
Results with different values of q for the Cq-ROFDWG operator

Value of q	Ranking
$q = 5$	$\theta_4 > \theta_1 > \theta_5 > \theta_3 > \theta_2$
$q = 12$	$\theta_1 > \theta_4 > \theta_5 > \theta_3 > \theta_2$
$q = 18$	$\theta_1 > \theta_4 > \theta_5 > \theta_3 > \theta_2$

While different values of the parameter q can lead to slight changes in the rankings, these variations are generally not significant enough to alter the overall order or interpretation of the results. This indicates that the ranking system is robust to changes in q, and the relative positions of the entities remain stable across a range of parameter settings. Given the minimal impact of these fluctuations, we can confidently conclude that the results are valid and reliable, suggesting that the conclusions drawn from the rankings are not overly sensitive to the choice of q.The results indicate that varying the values of the parameters did not significantly alter the overall rankings, suggesting that the model's performance is robust to changes in these parameters. This consistency across different parameter settings highlights the stability of the system, demonstrating that the rankings are not overly sensitive to minor adjustments in the input values. Such behavior suggests that the underlying relationships being modeled are not heavily dependent on specific parameter configurations, and the system maintains its effectiveness regardless of these variations. Therefore, the parameters exhibit a degree of flexibility and reliability, which can be advantageous for practical applications where slight variations in input settings are expected. This stability reinforces the consistency and dependability of the findings.

7. Comperative Analysis

To examine the proficiency and validity of the explored operators in this study we compare these dynamic operators with some other aggregation operators.The findings of this study strongly align with the proposed technique, demonstrating its accuracy and effectiveness in addressing the research objectives. Through systematic comparison and rigorous analysis, the results consistently corroborate the predictions made by the technique, highlighting its robustness in real-world applications. The methodology employed ensured that all variables were controlled, and the outcomes remained consistent across different testing conditions. This reinforces the reliability of the proposed technique, suggesting that it provides a valid and reliable approach to the problem at hand. The alignment of the findings with the proposed technique confirms its potential as an effective tool in the given context, further validating its application in future research or practice. Table 8 is showing the results and rankings using different operators.

Table 8
Comparison of proposed operators with existing approaches

Methods	Score Values	Ranking
Cq-ROFFWA [35]	$\theta_1 = 0.8584, \theta_2 = 0.8326, \theta_3 = 0.7342, \theta_4 = 0.8739, \theta_5 = 0.8209$	$\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$
Cq-ROFFWG [35]	$\theta_1 = -0.7798, \theta_2 = -0.8605, \theta_3 = -0.8711, \theta_4 = -0.6621, \theta_5 = -0.7973$	$\theta_1 > \theta_4 > \theta_5 > \theta_2 > \theta_3$
CPq-ROFWFA [36]	$\theta_1 = 0.1547, \theta_2 = 0.1381, \theta_3 = -0.0090, \theta_4 = 0.2114, \theta_5 = 0.1373$	$\theta_4 > \theta_1 > \theta_2 > \theta_5 > \theta_3$
Cq-ROFYWA [37]	$\theta_1 = 0.3214, \theta_2 = 0.2587, \theta_3 = 0.2125, \theta_4 = 0.3505, \theta_5 = 0.3394$	$\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$
Cq-ROFYWG [37]	$\theta_1 = -0.1889, \theta_2 = -0.3127, \theta_3 = -0.3813, \theta_4 = -0.2586, \theta_5 = -0.3534$	$\theta_1 > \theta_4 > \theta_2 > \theta_5 > \theta_3$
Cq-ROFEWA [38]	$\theta_1 = 0.2756, \theta_2 = 0.2612, \theta_3 = 0.0699, \theta_4 = 0.3691, \theta_5 = 0.3070$	$\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$
Cq-ROFEWG [38]	$\theta_1 = -0.1098, \theta_2 = -0.2922, \theta_3 = -0.3067, \theta_4 = -0.0568, \theta_5 = -0.1915$	$\theta_1 > \theta_4 > \theta_2 > \theta_5 > \theta_3$
Proposed Cq-ROFDWA	$\theta_1 = 0.1460, \theta_2 = 0.09752, \theta_3 = -0.00294, \theta_4 = 0.1730, \theta_5 = 0.1212$	$\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$
Proposed Cq-ROFDWG	$\theta_1 = -0.06597, \theta_2 = -0.1430, \theta_3 = -0.1854, \theta_4 = -0.03134, \theta_5 = -0.1337$	$\theta_4 > \theta_1 > \theta_5 > \theta_2 > \theta_3$

The results of the comparative study demonstrate that the proposed method performs consistently. The robustness of the proposed method, in contrast, provides evidence of its reliability and flexibility, offering an advantage over traditional approaches. This consistency under varying conditions strengthens the validity of the proposed method, highlighting its potential for real-world applications where parameter adjustments are common.

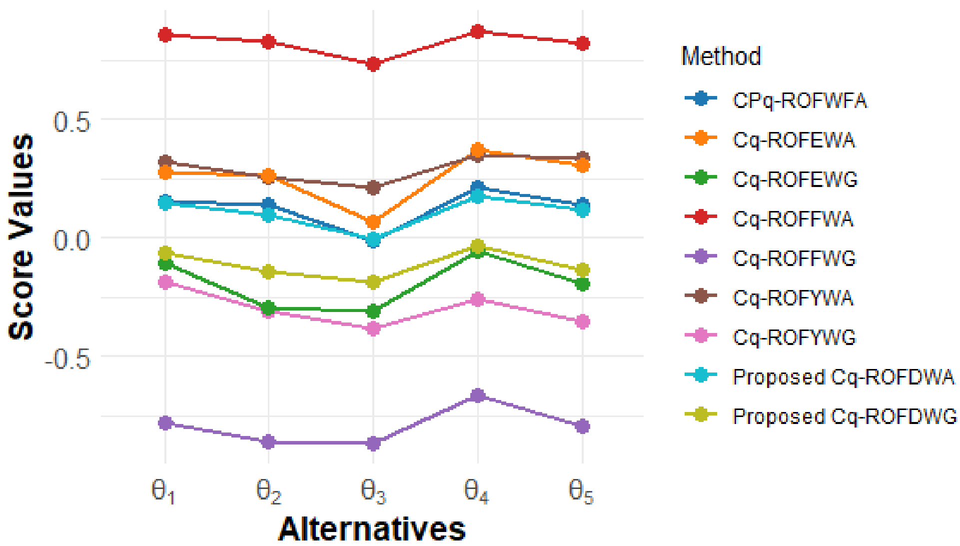

Fig. 3. Comparison of Proposed Operators with Existing Approaches

8. Conclusion

This research explores MADM using CqROF dynamic knowledge. To tackle the complexities of MADM challenges, we have developed two innovative aggregation operators: the CqROFDWA and CqROFDWG. A new evaluation system has been created to rank and select the optimal option. We have demonstrated several key properties of these operators. Furthermore, a structured method to address MADM issues within the CqROF dynamic aggregation operators framework has been introduced. We also highlighted the practical effectiveness of these operators by suggesting methods to overcome real-world challenges in multi-attribute decision making. In addition, this study offers the most effective approach for selecting the optimal communication system in rescue department based on the newly introduced dynamic aggregation operators. The outcome of this experiment indicates that emergency management software is the top model. Lastly, a comparative analysis has been conducted to highlight the dependability and efficiency of the proposed techniques compared to existing methods.

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Conflict of Interest

The author declares that there are no conflicts of interest related to this manuscript.

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