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Advanced Decision-Making with Complex q-Rung Orthopair Fuzzy Muirhead Mean Models

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ABSTRACT

Reliable and adaptable approaches are essential in MADM to address the complexities of medical diagnosis. To enhance decision-making (DM) processes, we propose a novel method combining the Muirhead mean (MM) with q-rung orthopair fuzzy sets (q-ROFS). The q-ROFS framework offers an advanced solution for handling ambiguity and vagueness in medical diagnoses by enabling a three-dimensional representation of expert opinions through membership degrees (MDs). The proposed method leverages MM's strength in capturing interrelationships among features, ensuring a more accurate and balanced integration of expert inputs. This integration is particularly valuable in medical diagnosis, where the interplay and relative importance of symptoms and diagnostic criteria are complex and critical. By expanding the theoretical applications of fuzzy sets (FS) in MADM, this innovative approach not only improves patient outcomes but also enhances the reliability of diagnostic procedures. It provides a practical tool for elevating DM quality in medical settings.

1. Introduction

Finding the most suitable possibilities with a set of attributes is the main objective of multiple-attribute decision-making (MADM), a prominent area within the field of decision science. Each possibility is assessed by a group of experts or an expert across various characteristics, and their ratings can be given either as a sharp number or another form. However, in the challenging world of today, it is typical for uncertainty to be a major factor in nearly every decision-making process. As a result, managing the analysis uncertainties is necessary. We must make decisions in various difficult contexts due to the growing complexity of economic systems. Representing attribute values appropriately is one of the most significant issues since decision-making (DM) is inherently fraught with fuzziness and uncertainty. Many tools have recently been created to describe and express vagueness and fuzziness.

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For example, Zadeh [1] introduced the fuzzy set (FS). DM based on FS has gained much attention due to its capacity to model uncertainty. FS has some drawbacks, and to overcome them, Atanassov [2] proposed the intuitionistic fuzzy set (IFS), which has two degrees: the degree of membership (MD) and the degree of non-membership (NMD). Because of this feature, it is more effective and potent than FS. Mahmood and Ali [3] used recently established techniques to modify the theory of complex intuitionistic fuzzy sets (CIFSs) to define interrelationships among various arguments throughout the aggregate phase. Peng and Dai [4] studied the use of Pythagorean fuzzy (PyF) uncertain linguistic information and Pythagorean 2-tuple linguistic information, respectively, for DM tasks. Singh [5] suggested the PyF set (PyFS) correlation coefficient and used it in clustering analysis. When describing uncertain information in subjective DM processes, Yager *et al.,* [6] introduced a q-rung orthopair fuzzy set (q-ROFS). Ramot *et al.,* [7] proposed the theory of complex FSs (CFSs) by extending the MD range to the unit circle in the complex plane. Alkouri and Salleh [8] introduced the CIFS, wherein polar coordinates over the unit disc denote the complex-valued MD and complex-valued NMD for each element. Kumar and Bajaj [9] first proposed the concept of distance measurements and entropy for CIFSs. Yager and Abbasov [10] examined the connection between complex and Pythagorean numbers.

Many researchers have offered various approaches for resolving MADM problems through hybrid operators, similarity or distance measurements, aggregation operators (AOs), and other techniques. Wang and Garg [11] developed a method that uses the properties of Archimedean triangular norms to manage ambiguous data in MADM. Xu and Yager [12] presented a technique that uses the Bonferroni mean to combine intuitionistic fuzzy (IF) data, improving the system's capacity to manage uncertainty. Verma and Sharma [13] introduced a new method for evaluating inaccuracy between IFSs and applied it to IF MADM. Dengfeng and Chuntian [14] established the IFN similarity metrics. Garg and Kumar [15] used IFN connection numbers to create similarity metrics for IFNs. Fuping *et al.,* [16] expanded the theory of FSs to include Pythagorean triangular FSs and hesitant probabilistic FSs. They also presented novel methods based on PyF information to solve multi-criteria decision-making (MCDM) problems. Yager [17] defined the Pythagorean fuzzy numbers (PyFNs) weighted average of AOs. Riaz and Farid [18] created a novel AO structure based on SF conditions to handle unclear and complex data in computational and intelligent systems. Yu [19] expanded the IF environment by using the classical Heronian mean (HM) and created several intuitionistic HM operators with fuzzy attributes. Wei *et al.,* [20] suggested using the q-ROFS to create the HM operator to solve MADM difficulties. Rani and Garg [21] developed power and weighted AOs to address MADM challenges. Calvo *et al.,* [22] generated theoretical notions for weighted geometric operators and weighted averages. He *et al.,* [23] proposed the Intuitionistic Fuzzy Power Geometric Bonferroni Mean (IFPGBM), which combines the power geometric mean with the Bonferroni mean within the IFS framework. This approach enhances DM procedures by integrating both MD and NMD in multiple-attribute group decision-making (MAGDM) situations. Ullah *et al.,* [24] developed aggregation techniques to overcome ambiguity and imprecision in information when making decisions in a complex PyF (CPyF) setting. Kumar and Chen [25] presented new AOs to address the shortcomings of previous methods by applying the complex theory of linguistic information for IF and developed a multi-criteria group decision-making (MCGDM) method to resolve practical problems. Hussain *et al.,* [26] proposed innovative methods for applications under the MCDM technique using Aczel-Alsina (AA) aggregation tools based on PyF information. Naeem and Ali [27] categorized renewable energy sources using bipolar complex fuzzy theory and Frank aggregation methods. Garg [28] examined the uses of trigonometric functions in scenarios involving repeated data. These mathematical techniques are crucial for decision support systems based on our analysis of various strategies. Ahmmad [29] presents novel entropy measurements for fuzzy soft sets with q-rung orthopairs. Ali [30] investigates sophisticated IF power interaction AOs for advanced DM processes. In an effort to investigate the Dombi aggregation operators in the context of bipolar complex fuzzy soft information, Jaleel [31] incorporated the WASPAS technique, a popular MCDM approach for robotic system optimization in agriculture that evaluates and ranks operational strategies under uncertainty. Chai *et al.,* [32] proposed a method combining IFSs and interval-valued fuzzy sets (IVFSs) to handle imprecision and uncertainty in data. Garg [33] offered PyFN neutrality AOs to address MADM issues. Nie *et al.,* [34] proposed divided Bonferroni mean AOs using the Shapley fuzzy measure to solve DMPs. Liu and Wang [35] examined geometric operators and the q-rung fuzzy weighted average. Garg and Chen [36] created q-ROFS neutrality AOs to address MADM issues. We also studied mathematical approaches and advanced decision-making methodologies in [37–40].

Although methods for comprehending complex human viewpoints are useful, decision-makers frequently encounter unforeseen difficulties when conducting assessments. Researchers have integrated three fundamental concepts with new mathematical strategies to control complexity. These techniques perform well when dealing with partial data that does not use weighted attributes. The main goal of this work is to explore mathematical strategies within the Complex q-rung orthopair fuzzy Muirhead mean (Cq-ROFMM) framework. The following are some advantages of the derived theory:

a) Complex q-rung Orthopair Fuzzy Sets (Cq-ROFS) enable more advanced handling of imprecision and uncertainty in DM procedures. This is especially helpful for medical diagnosis since symptoms and data are frequently unclear or lacking.
b) The methodology offers a more accurate and dependable evaluation of numerous qualities by integrating Cq-ROFS. Making more educated judgments is the result, and this is crucial when it comes to identifying medical issues.
c) Transparency is improved by the methodical approach provided by the methodology. In medical settings, where accountability and transparency are crucial, decision-makers must be able to clearly comprehend and justify their decisions.
d) The theory can be customized to fit diverse medical diagnostic circumstances since it can be adjusted to different choice contexts and preferences. Because of its adaptability, it is a useful instrument that may be used to address a variety of medical issues.
e) The methodology permits the combination of several standards and their mutual connections, so simplifying an all-encompassing assessment of possible diagnoses. This comprehensive approach guarantees that all relevant factors are taken into account, resulting in more reliable diagnostic results.
f) MAGDM is supported by the methodology, which is especially helpful in medical settings where collaborative DM is typical. It guarantees the efficient integration of the knowledge and experience of many stakeholders.
g) The theory facilitates optimal medical diagnosis selection and evaluation by utilizing Cq-ROFS and Muirhead methods. Better patient outcomes result from ensuring that the selected diagnosis complies with the particular needs and preferences of the medical setting.

The remaining sections of the manuscript are organised as follows: Section 2 provides a fundamental overview of Cq-ROFSs, comparison rules and MM operators. We designed a list of innovative mathematical approaches for MM operators under the system of Cq-ROFVs in section 3. Section 4 also developed AOs of DMM operators. Section 5 presented a comprehensive overview of the decision-making problem with the experimental case study. To show the validation of pioneered AOs, the comparison method is established with different existing mathematical approaches in section 6. Section 7 summarises the whole article with limitations and future directions.

2. Preliminaries

In this section, we overview basic notions of fuzzy frameworks with fundamental operations.

Definition 1: [6] A q-ROFS on a universal set X is given by:

$$R = \{(x, u(x), v(x)): x \in X\}$$

Where $u: X \rightarrow [0, 1]$ and $v: X \rightarrow [0, 1]$ indicate, the MD and the NMD, respectively with a condition: $0 \leq u^q(x) + v^q(x) \leq 1 \, and \, q > 0$. The HD is denoted by $\theta_R = \sqrt[q]{(1 - u^q(x) - v^q(x))}$, and the term $R = (u(x), v(x))$ represents the q-ROFVs.

Definition 2: [7] A CFS on a universal set X is defined as follows:

$$R = \{(x, u(x))|, x \in X\}$$

Where $u(x)e^{i2\pi(\phi)}$ such that $u(x) \in [0, 1]$ and $\phi(x) \in [0,1]$ indicates the real term of the MD and phase term of the MD, respectively with the conditions:

$$0 \leq \phi(x) \leq 1 \, and \, 0 \leq \psi(x) \leq 1.$$

Definition 3: [8] A CIFS on a universal set X is given by:

$$R = \{(x, u(x), v(x)): x \in X\}$$

Where $u(x)e^{i2\pi(\phi)}$ and $v(x)e^{i2\pi(\psi)}$ indicate the MD and NMD on a CIFS. Furthermore, $u(x)e^{i2\pi(\phi)}$ such that $u(x) \in [0, 1]$ and $\phi(x) \in [0,1]$ indicates the real term of the MD and phase term of the MD, respectively with subject to the conditions:

$$0 \leq u(x) + v(x) \leq 1, 0 \leq \phi(x) + \psi(x) \leq 1.$$

Definition 4: [24] A C-PyFS on a universal set X is defined as follows:

$$R = \{(x, u(x), v(x)): x \in X\}$$

Where $u(x)e^{i2\pi(\phi)}$ and $v(x)e^{i2\pi(\psi)}$ are MD and NMD on CIFS. Furthermore, $u(x)e^{i2\pi(\phi)}$ such that $u(x) \in [0, 1]$ and $\phi(x) \in [0,1]$ indicates the real term of the MD and phase term of the MD. Similarly, v and ψ represent the real term of NMD and phase term of NMD, respectively and subject to the conditions:

$$0 \leq u^2(x) + v^2(x) \leq 1, \text{and } 0 \leq \phi^2(x) + \psi^2(x) \leq 1.$$

Additionally, the HD of x is represented by $\theta_R = \sqrt{1 - (u^2(x) + v^2(x))}$.

Definition 5: [41] A Cq-ROFS on a universal set X is defined as follows:

$$R = \{(x, u(x), v(x)): x \in X\}$$

Where $u(x)e^{i2\pi(\phi)}$ and $v(x)e^{i2\pi(\psi)}$ are MD and NMD on CIFS. Furthermore, $u(x)e^{i2\pi(\phi)}$ such that $u(x) \in [0, 1]$ and $\phi(x) \in [0,1]$ indicates the real term of the MD and phase term of the MD respectively. Similarly, v and ψ represent the real term and phase term of NMD respectively. A Cq-ROFS must satisfy the following conditions:

$$0 \leq u^q(x) + v^q(x) \leq 1, \text{and } 0 \leq \phi^q(x) + \psi^q(x) \leq 1. \, q > 0$$

The Cq-ROFV is represented by $R = (u(x)e^{i2\pi(\phi)}, v(x)e^{i2\pi(\psi)})$.

Definition 6: [42] For a Cq-ROFV $R_1 = (u_1(x)e^{i2\pi(\phi_1)} + v_1(x)e^{i2\pi(\psi_1)})$. The score function $S(R_1)$ and accuracy function $H(R_1)$ are described as:

$$S(R_1) = \frac{1}{4}\left(1 + \left(u_1^q - v_1^q\right) + \left(\phi_1^q - \psi_1^q\right)\right)$$

$$H(R_1) = u_1^q + v_1^q + \phi_1^q + \psi_1^q$$

Theorem 1: [42] Consider two Cq-ROFVs R_1 and R_2 with any positive real numbers γ_1 and γ_2. Some properties are also discussed as follows:

1. $\gamma_1(R_1 \oplus R_2) = \gamma_1 R_1 \oplus \gamma_1 R_2$
2. $(R_1 \otimes R_2)^{\gamma_1} = R_1^{\gamma_1} \otimes R_2^{\gamma_1}$
3. $(\gamma_1 + \gamma_2)R_1 = \gamma_1 R_1 + \gamma_2 R_1$
4. $R_1^{\gamma_1+\gamma_2} = R_1^{\gamma_1} \otimes R_2^{\gamma_1}$

Proof: We look at parts (1) and (3); the rest are rather simple. $a_\varpi = \left(u_\varpi e^{2i\pi(\phi_\varpi)}, v_\varpi e^{2i\pi(\psi_j)}\right)(\varpi = 1,2,3,\dots,m)$

(1) Let $R_1 = \left(u_1(x)e^{i2\pi(\phi_1)} + v_1(x)e^{i2\pi(\psi_1)}\right)$ and $R_2 = \left(u_2(x)e^{i2\pi(\phi_2)} + v_2(x)e^{i2\pi(\psi_2)}\right)$ be two Cq-ROFVs. Then

$$\gamma_1(R_1 \oplus R_2) = \gamma_1 \left(\begin{array}{c} \left(1 - \prod_{\varpi=1}^{2}(1 - (u_\varpi)^q)\right)^{\frac{1}{q}} e^{2\pi i\left(\left(1 - \prod_{\varpi=1}^{2}(1-(\phi_\varpi)^q)\right)^{\frac{1}{q}}\right)}, \\ \prod_{\varpi=1}^{2} v_\varpi \, e^{2\pi i\left(\prod_{\varpi=1}^{2}\psi_\varpi\right)} \end{array} \right)$$

$$= \left(\begin{array}{c} \left(1 - \prod_{\varpi=1}^{2}(1 - (u_\varpi)^q)^{\gamma_1}\right)^{\frac{1}{q}} e^{2\pi i\left(\left(1 - \prod_{\varpi=1}^{2}(1-(\phi_\varpi)^q)^{\gamma_1}\right)^{\frac{1}{q}}\right)}, \\ \left(\prod_{\varpi=1}^{2} v_\varpi\right)^{\gamma_1} e^{2\pi i\left(\prod_{\varpi=1}^{2}\psi_\varpi\right)^{\gamma_1}} \end{array} \right)$$

$$= \left((1 - (1 - (u_1)^q)^{\gamma_1})^{\frac{1}{q}} e^{2\pi i\left(\left(1 - \left(1 - (\phi_1^q)^{\gamma_1}\right)\right)^{\frac{1}{q}}\right)}, (v_1)^{\gamma_1} e^{2\pi i(\psi_1)^{\gamma_1}} \right)$$

$$\oplus \left((1 - (1 - (u_2)^q)^{\gamma_1})^{\frac{1}{q}} e^{i2\pi\left(\left(1 - \left(1 - (\phi_2^q)^{\gamma_1}\right)\right)^{\frac{1}{q}}\right)}, (v_2)^{\gamma_1} e^{i2\pi(\psi_2)^{\gamma_1}} \right)$$

$$= \gamma_1 R_1 + \gamma_1 R_2$$

(2) Let $R_1 = \left(u_1(x)e^{i2\pi(\phi_1)} + v_1(x)e^{i2\pi(v_1)}\right)$ and $R_2 = \left(u_2(x)e^{i2\pi(\phi_2)} + v_2(x)e^{i2\pi(\psi_1)}\right)$ be two Cq-ROFVs. Then

$$(\gamma_1 + \gamma_2)R_1 = (\gamma_1 + \gamma_2)\left(u_1(x)e^{i2\pi(\phi_1)} + v_1(x)e^{i2\pi(\psi_1)}\right)$$

$$= \left(\left(1 - (1 - (u_1)^q)^{(\gamma_1+\gamma_2)}\right)^{\frac{1}{q}} e^{i2\pi\left(\left(1 - \left(1 - (\phi_1^q)^{(\gamma_1+\gamma_2)}\right)\right)^{\frac{1}{q}}\right)}, (v_1)^{(\gamma_1+\gamma_2)} e^{i2\pi(\psi_1)^{(\gamma_1+\gamma_2)}} \right)$$

$$= \left((1-(1-(u_1)^q)^{\gamma_1})^{\frac{1}{q}} e^{i2\pi\left(\left(1-\left(1-(\phi_1^q)^{\gamma_1}\right)\right)^{\frac{1}{q}}\right)}, (v_1)^{\gamma_1} e^{i2\pi(\psi_1)^{\gamma_1}} \right)$$

$$\oplus \left((1-(1-(u_1)^q)^{\gamma_2})^{\frac{1}{q}} e^{i2\pi\left(\left(1-\left(1-(\phi_2^q)^{\gamma_2}\right)\right)^{\frac{1}{q}}\right)}, (v_1)^{\gamma_2} e^{i2\pi(\psi_1)^{\gamma_2}} \right)$$

$$= \gamma_1 R_1 + \gamma_2 R_1$$

Hence $(\gamma_1 + \gamma_2)R_1 = \gamma_1 R_1 + \gamma_2 R_1$.

First of all, the theoretical concepts of Muirhead Mean (MM) operator was proposed by Muirhead [43]. Unlike traditional methods that treat each number independently, its main advantage is its capacity to recognize and take into account the links between the data being aggregated. Because of this, MM is especially helpful in domains where the relationships between components are essential to the result, such as DM and data aggregation.

Definition 7: [43] Let a_ϖ $(\varpi = 1,2,3,\ldots,m)$ be a collection of crisp values and $P = (p_1, p_2, \ldots, p_m) \in R^m$ be a vector of parameters. Then, the MM is defined as

$$MM^P(a_1, a_2, \ldots, a_m) = \left(\frac{1}{m!}\sum_{\vartheta \in S_m} \prod_{\varpi=1}^{m} a_{\vartheta(\varpi)}^{P_\varpi}\right)^{\frac{1}{\sum_{\varpi=1}^{m} P_\varpi}}$$

where $\vartheta(\varpi)$ $(\varpi = 1,2,3,\ldots,m)$ is any permutation of $(1,2,3,\ldots,m)$, and S_m is the gathering of all permutations of $(1,2,3,\ldots,m)$.

Definition 8: [44] Let a_ϖ $(\varpi = 1,2,3,\ldots,m)$ be a collection of crisp values and $P = (p_1, p_2, \ldots, p_m) \in R^m$ is a vector of parameters. Then, the DMM is specified as

$$DMM^P(a_1, a_2, \ldots, a_m) = \frac{1}{\sum_{\varpi=1}^{n} P_\varpi}\left(\prod_{\vartheta \in S_m} \sum_{\varpi=1}^{m} \left(P_\varpi a_{\vartheta(\varpi)}\right)\right)^{\frac{1}{m!}}$$

where $\vartheta(\varpi)$ $(\varpi = 1,2,3,\ldots,m)$ is any permutation of $(1,2,3,\ldots,m)$, and S_m is the gathering of all permutations of $(1,2,3,\ldots,m)$.

3. Muirhead Mean Operators Based on Complex q-Rung Orthopair Fuzzy Information

Definition 9: Let $a_\varpi = \left(u_\varpi e^{2i\pi(\phi_\varpi)}, v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi = 1,2,3,\ldots,m)$ be a collection of Cq-ROFVs, $P = (p_1, p_2, \ldots, p_m) \in R^m$ be a vector of parameters, then the Cq-ROFMM operator is specified as

$$Cq-ROFMM^P(a_1, a_2, \ldots, a_m) = \left(\frac{1}{m!}\sum_{\vartheta \in S_m} \prod_{\varpi=1}^{m} a_{\vartheta(\varpi)}^{P_\varpi}\right)^{\frac{1}{\sum_{\varpi=1}^{m} P_\varpi}}$$

where $\vartheta(\varpi)(\varpi = 1,2,3,\ldots,m)$ is any permutation of $(1,2,3,\ldots,m)$, and S_m is the collection of all permutations of $(1,2,3,\ldots,m)$.

Theorem 2: Let $a_\varpi = \left(u_\varpi e^{2i\pi(\phi_\varpi)}, v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi = 1,2,3,\ldots,m)$ be a collection of Cq-ROFVs, $P = (p_1, p_2, \ldots, p_m) \in R^m$ be a vector of parameters, in which case the Cq-ROFMM operator aggregated value is still a Cq-ROFVs and we have:

$$q-ROFMM^{P}\,(a_1,a_2,\dots,a_m)=\left(\begin{array}{c}\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)},\\ \left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}} e^{2\pi i\left(\left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}}\right)}\end{array}\right)$$

Proof: We have:

$$a_{\vartheta(\varpi)}^{P_\varpi}=\left(u_{\vartheta(\varpi)}^{qP_\varpi}e^{2\pi i\left(\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)},\left(1-\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}e^{2\pi i\left(\left(1-\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}\right)}\right)$$

$$\prod_{\varpi=1}^{m}a_{\vartheta(\varpi)}^{P_\varpi}=\left(\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{qP_\varpi}\,e^{2\pi i\left(\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)},\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}e^{2\pi i\left(\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}\right)}\right)$$

Therefore,

$$\sum_{\vartheta\in S_m}\prod_{\varpi=1}^{m}a_{\vartheta(\varpi)}^{P_\varpi}=\left(\begin{array}{c}\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{q}}e^{2\pi i\left(\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{q}}\right)},\\ \prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}e^{2\pi i\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}\right)}\end{array}\right)$$

And

$$\left(\frac{1}{m!}\sum_{\vartheta\in S_m}\prod_{\varpi=1}^{m}a_{\vartheta(\varpi)}^{P_\varpi}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}$$

$$=\left(\begin{array}{c}\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)},\\ \left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}\right)^{\frac{1}{m!}} e^{2\pi i\left(\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{q}}\right)^{\frac{1}{m!}}\right)}\end{array}\right)$$

Thus,

$$\left(\frac{1}{m!}\sum_{\vartheta\in S_m}\prod_{\varpi=1}^{m}a_{\vartheta(\varpi)}^{P_\varpi}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}=\left(\begin{array}{c}\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\\ e^{2\pi i\left(\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)},\\ \left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}}\\ e^{2\pi i\left(\left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}}\right)}\end{array}\right)$$

Further, let

$$u=\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)}$$

And

$$v=\left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}}$$

$$e^{2\pi i\left(\left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}}\right)}$$

It is easy to prove that $0 \leq u \leq 1$ and $0 \leq v \leq 1$.

Since $u_{\vartheta(\varpi)}^{q} + v_{\vartheta(\varpi)}^{q} \leq 1$, then $u_{\vartheta(\varpi)}^{q} \leq 1 - v_{\vartheta(\varpi)}^{q}$, we can get

$$u^q + v^q = \left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m} u_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{qP_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)}$$

$$+1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-u_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\phi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)} \leq$$

$$\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)}$$

$$+1-\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)}$$

$$= 1$$

It indicates that a Cq-ROFV is also the total value determined by the Cq-ROFMM operator. Consequently, Theorem 1's proof is proved.

Case 1: If $P = (1, 0, 0, \ldots, 0)$, subsequently, operator Cq-ROFMM shrinks to the following:

$$Cq-ROFMM^{(1,0,0,\dots,0)}(a_1,a_2,\dots,a_m)=\left(\begin{array}{c}\left(1-\prod_{\varpi=1}^{m}\left(1-u_{\varpi}^{q}\right)^{\frac{1}{m}}\right)^{\frac{1}{q}}e^{2\pi i\left(\left(1-\prod_{\varpi=1}^{m}\left(1-\phi_{\varpi}^{q}\right)^{\frac{1}{m}}\right)^{\frac{1}{q}}\right)},\\ \prod_{\varpi=1}^{m}v_{\varpi}^{\frac{1}{m}}e^{2\pi i\left(\prod_{\varpi=1}^{m}\psi_{\varpi}^{\frac{1}{m}}\right)}\end{array}\right)$$

$$=\frac{1}{m}\sum_{\varpi=1}^{m}a_{\varpi}$$

Case 2: If $P=(\lambda,0,0,\dots,0)$, subsequently, operator Cq-ROFMM shrinks to the following:

$$Cq-ROFMM^{(\lambda,0,0,\dots,0)}(a_1,a_2,\dots,a_m)$$

$$=\left(\begin{array}{c}\left(1-\left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-u_{\varpi}^{\lambda}\right)\right)\right)^{\lambda}\right)^{\frac{1}{q\lambda}}e^{2\pi i\left(\left(1-\left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-\phi_{\varpi}^{\lambda}\right)\right)\right)^{\lambda}\right)^{\frac{1}{q\lambda}}\right)},\\ \left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-\left(1-v_{\varpi}^{q}\right)^{\lambda}\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right)^{\frac{1}{q}}e^{2\pi i\left(\left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-\left(1-\psi_{\varpi}^{q}\right)^{\lambda}\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right)^{\frac{1}{q}}\right)}\end{array}\right)$$

$$=\left(\frac{1}{m}\sum_{\varpi=1}^{m}a_{\varpi}^{\lambda}\right)^{\frac{1}{\lambda}}$$

It is the Liu and Wang-defined Cq-rung orthopair fuzzy generalized arithmetic averaging (Cq-ROFA) operator.

Case 3: If $P=(1,1,0,0,\dots,0)$, subsequently, operator Cq-ROFMM shrinks to the following:

$$Cq-ROFMM^{(1,1,0,0\dots,0)}(a_1,a_2,\dots,a_m)$$

$$=\left(\begin{array}{c}\left(1-\left(\prod_{\substack{i,\varpi=1\\ i\neq\varpi}}\left(1-\left(u_iu_{\varpi}\right)^{q}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{2q}}\\ e^{2\pi i\left(\left(1-\left(\prod_{\substack{i,\varpi=1\\ i\neq\varpi}}\left(1-\left(\phi_i\phi_{\varpi}\right)^{q}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{2q}}\right)},\\ \left(1-\left(1-\left(\prod_{\substack{i,\varpi=1\\ i\neq\varpi}}^{m}\left(v_i^{q}+v_{\varpi}^{q}-v_i^{q}v_{\varpi}^{q}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q}}\\ e^{2\pi i\left(\left(1-\left(1-\left(\prod_{\substack{i,\varpi=1\\ i\neq\varpi}}^{n}\left(\psi_i^{q}+\psi_{\varpi}^{q}-\psi_i^{q}\psi_{\varpi}^{q}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q}}\right)}\end{array}\right)$$

$$=\left(\frac{1}{m(m-1)}\sum_{\substack{i,\varpi=1\\ i\neq\varpi}}^{m} a_i a_\varpi\right)^{\frac{1}{2}}$$

Case 4: When $P=\overbrace{(1,1,1,\ldots,1)}^{k},\overbrace{(0,0,0,\ldots,0)}^{m-k}$, subsequently, operator Cq-ROFMM shrinks to the following:

$$Cq-ROFMM^{\overbrace{(1,1,1,\ldots,1)}^{k},\overbrace{(0,0,0,\ldots,0)}^{m-k}}(a_1,a_2,\ldots,a_m)$$

$$=\left(\begin{array}{c}\left(\left(1-\left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-\left(\prod_{\varpi=1}^{m}\mu_{i\varpi}\right)^{q}\right)\right)\right)\right)^{\frac{1}{C_m^k}}\right)^{\frac{1}{qk}} e^{2\pi i\left(\left(1-\left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-\left(\prod_{\varpi=1}^{m}\phi_{i\varpi}\right)^{q}\right)\right)\right)^{\frac{1}{C_m^k}}\right)^{\frac{1}{qk}}\right)},\\ \left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-\prod_{\varpi=1}^{m}(1-v_{i\varpi})\right)^{\frac{1}{C_m^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}} e^{2\pi i\left(\left(1-\left(1-\prod_{\varpi=1}^{m}\left(1-\prod_{\varpi=1}^{m}(1-\psi_{i\varpi})\right)^{\frac{1}{C_m^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}}\right)}\end{array}\right)$$

$$=\left(\frac{\bigoplus_{1\le i_1\le\cdots\le i_k\le m}\bigoplus_{\varpi=1}^{m} a_{i_\varpi}}{c_m^k}\right)^{\frac{1}{q}}$$

It refers to the operator Cq-rung orthopair fuzzy Maclaurin symmetric mean (Cq-ROFMSM).

Case 5: If $P=(1,1,1,\ldots,1)$, subsequently, operator Cq-ROFMM shrinks to the following:

$$Cq-ROFMM^{(1,1,1,\ldots,1)}(a_1,a_2,\ldots,a_m)=\left(\begin{array}{c}\prod_{\varpi=1}^{n}u_{\varpi}^{\frac{1}{m}}e^{2\pi i\left(\prod_{\varpi=1}^{m}\phi_{\varpi}^{\frac{1}{m}}\right)},\\ \prod_{\varpi=1}^{m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\varpi}^{q}\right)^{\frac{1}{m}}\right)^{\frac{1}{q}} e^{2\pi i\left(\prod_{\varpi=1}^{m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\varpi}^{q}\right)^{\frac{1}{m}}\right)^{\frac{1}{q}}\right)}\end{array}\right)$$

$$=\prod_{\varpi=1}^{m}a_{\varpi}^{\frac{1}{m}}$$

The above operator means Cq-rung orthopair fuzzy geometric averaging (Cq-ROFG), proposed by Liu and Wang.

Case 6: If $P=(1/m,1/m,1/m,\ldots,1/m)$, subsequently, operator Cq-ROFMM shrinks to the following:

$$Cq-ROFMM^{(1/m,1/m,1/m,\ldots,1/m)}(a_1,a_2,\ldots,a_m)$$

It is the Liu and Wang-defined Cq-rung operator (Cq-ROFG).

Case 7: If $q=2$, subsequently, operator Cq-ROFMM shrinks to the following:

$$Cq-ROFMM^{P}\left(a_1,a_2,\dots,a_m\right)=\left(\begin{array}{c}\left(\sqrt{1-\left(\prod_{\vartheta\in S_m}\left(1-\left(\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{P_\varpi}\right)^{2}\right)\right)^{\frac{1}{m!}}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(\sqrt{1-\left(\prod_{\vartheta\in S_m}\left(1-\left(\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{P_\varpi}\right)^{2}\right)\right)^{\frac{1}{m!}}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)},\\ \sqrt{1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}^{2}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}} e^{2\pi i\left(\sqrt{1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}^{2}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}}\right)}\end{array}\right)$$

The operator in the above discussion is the Pythagorean fuzzy Muirhead mean (Py-FMM).

Case 8: If $q=1$, subsequently, the Cq-ROFMM shrinks to the following

$$Cq-ROFMM^{p}\left(a_1,a_2,\dots,a_m\right)=\left(\begin{array}{c}\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}u_{\vartheta(\varpi)}^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\phi_{\vartheta(\varpi)}^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)},\\ 1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-v_{\vartheta(\varpi)}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\psi_{\vartheta(\varpi)}\right)^{P_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)}\end{array}\right)$$

It is the Liu and Li defined intuitionistic fuzzy Muirhead mean (IFMM).

The following characteristics of the Cq-ROFMM are easily proven.

Theorem 3: Let $a_\varpi=\left(u_\varpi e^{2i\pi(\phi_\varpi)},v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi=1,2,3,\dots,m)$ be a group of Cq-ROFVs if all the Cq-ROFVs are equal, i.e. $a_\varpi=a=(u,v)$, then

$$Cq-ROFMM^{p}\left(a_1,a_2,\dots,a_m\right)=a$$

Theorem 4: Let $a_\varpi=\left(u_\varpi e^{2i\pi(\phi_\varpi)},v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi=1,2,3,\dots,m)$ and $b_j=(s_\varpi,t_\varpi)(\varpi=1,2,3,\dots,m)$ be two gatherings of Cq-ROFVs. If $u_\varpi\geq s_\varpi$ and $v_\varpi\leq t_\varpi$ for all i then

$$Cq-ROFMM^{p}\left(a_1,a_2,\dots,a_m\right)\geq Cq-ROFMM^{p}\left(b_1,b_2,\dots,b_m\right)$$

Theorem 5: Let $a_\varpi=\left(u_\varpi e^{2i\pi(\phi_\varpi)},v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi=1,2,3,\dots,m)$ be a gathering of Cq-ROFVs, then:

$$a^{-}\leq Cq-ROFMM^{p}\left(a_1,a_2,\dots,a_m\right)\leq a^{+}$$

Where $a^{+}=\left(max_{\varpi=1}^{m}(u_\varpi),min_{\varpi=1}^{m}(v_\varpi)\right)$ and $a^{-}=\left(min_{\varpi=1}^{m}(u_\varpi),max_{\varpi=1}^{m}(v_\varpi)\right)$

Definition 10: Let $a_\varpi = (u_\varpi, v_\varpi)(\varpi = 1,2,3, \dots, m)$ be a collection of Cq-ROFVs $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ be the weight vector of $a_\varpi(\varpi = 1,2,3, \dots, m)$, fulfilling $\omega_i \in [0,1]$ and $\sum_{i=1}^{m} \omega_i = 1$, and let $P = (p_1, p_2, \dots, p_m) \in R^m$ be a vector of parameters. If

$$Cq - ROFWMM^p\ (a_1, a_2, \dots, a_m) = \left(\frac{1}{m!} \sum_{\vartheta \in S_m} \prod_{\varpi=1}^{m} \left(m\omega_{\vartheta(\varpi)} a_{\vartheta(\varpi)}\right)^{P_\varpi}\right)^{\frac{1}{\sum_{\varpi=1}^{m} P_\varpi}}$$

then $Cq - ROFWMM^P$ is the $Cq - ROFWMM$, and $\vartheta\,(\varpi)\,(\varpi = 1, 2,3, \dots, m)$ is the permutation of $(1, 2, 3, \dots, m)$, and S_m is the group of all permutations of $(1, 2,3, \dots, m)$.

Theorem 6: Let $a_\varpi = (u_\varpi, v_\varpi)(\varpi = 1,2,3, \dots, m)$ be a group of Cq-ROFVs and $P = (p_1, p_2, \dots, p_m) \in R^m$ is a vector of parameters, then the Cq-ROFV is also the aggregated value produced by the Cq-ROFWMM and

$$Cq - ROFWMM^P\ (a_1, a_2, \dots, a_m)$$

$$= \left(\begin{array}{c} \left(\left(1 - \left(\prod_{\vartheta \in S_m}\left(1 - \prod_{\varpi=1}^{m}\left(1 - \left(1 - u_{\vartheta(\varpi)}^{q}\right)^{m\omega_{\vartheta(\varpi)}}\right)^{P_i}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m} P_\varpi}} e^{2\pi i\left(\left(\left(1 - \left(\prod_{\vartheta \in S_m}\left(1 - \prod_{\varpi=1}^{m}\left(1 - \left(1 - \phi_{\vartheta(\varpi)}^{q}\right)^{m\omega_{\vartheta(\varpi)}}\right)^{P_i}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m} P_\varpi}}\right)}, \\ \left(1 - \left(1 - \left(\prod_{\vartheta \in S_m}\left(1 - \prod_{\varpi=1}^{m}\left(1 - \left(v_{\vartheta(\varpi)}^{m\omega_{\vartheta(\varpi)}}\right)^{q}\right)^{P_i}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m} P_\varpi}}\right)^{\frac{1}{q}} e^{2\pi i\left(\left(1 - \left(1 - \left(\prod_{\vartheta \in S_m}\left(1 - \prod_{\varpi=1}^{m}\left(1 - \left(\psi_{\vartheta(\varpi)}^{n\omega_{\vartheta(\varpi)}}\right)^{q}\right)^{P_i}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m} P_\varpi}}\right)^{\frac{1}{q}}\right)} \end{array} \right)$$

4. Dual Muirhead Mean Operators Based on Complex q-Rung Orthopair Fuzzy Information

In this section, we define the Cq-ROFDMM operations are the weighted AOs for the collections of Cq-ROFVs.

Definition 11: Let $a_\varpi = \left(u_\varpi e^{2i\pi(\phi_\varpi)}, v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi = 1,2,3, \dots, m)$ be a collection of Cq-ROFVs and $P = (p_1, p_2, \dots, p_m) \in R^m$ be a vector of parameter, then the Cq-ROFWMM is defined as

$$Cq - ROFDMM^P\ (a_1, a_2, \dots, a_m) = \frac{1}{\sum_{\varpi=1}^{m} P_\varpi}\left(\prod_{\vartheta \in S_m} \sum_{\varpi=1}^{m} \left(P_\varpi a_{\vartheta(\varpi)}\right)\right)^{\frac{1}{m!}}$$

where $\vartheta\,(\varpi)\,(= 1, 2,3, \dots, m)$ is the permutation of $(1, 2,3, \dots, m)$, and S_m is the groups of all permutations of $(1, 2, \dots, m)$.

In the same way, above Definition leads to the following theorem.

Theorem 7: Let $a_\varpi = \left(u_\varpi e^{2i\pi(\phi_\varpi)}, v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi = 1,2,3, \dots, m)$ be a gathering of Cq-ROFNs and $P = (p_1, p_2, \dots, p_m) \in R^m$ is a vector of parameters, in which case the Cq-ROFDMM operator aggregated value is still a Cq-ROFVs and

$$Cq-ROFDMM^{P}\,(a_1,a_2,\dots,a_m)=\left(\begin{array}{c}\left(\left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-u_{\vartheta(\varpi)}^{q}\right)^{P_i}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}}\right) e^{2\pi i\left(\left(1-\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\phi_{\vartheta(\varpi)}^{q}\right)^{P_i}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{\frac{1}{q}}\right)},\\ \left(\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}v_{\vartheta(\varpi)}^{qp_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right) e^{2\pi i\left(\left(\left(1-\left(\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\psi_{\vartheta(\varpi)}^{qp_\varpi}\right)\right)^{\frac{1}{m!}}\right)^{\frac{1}{q}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)}\end{array}\right)$$

The proof of *Theorem 7* is similar to that of *Theorem 1*.

Definition 12: Let $a_\varpi=\left(u_\varpi e^{2i\pi(\phi_\varpi)},v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi=1,2,3,\dots,m)$ be a gathering of Cq-ROFVs $\omega=(\omega_1,\omega_2,\dots,\omega_m)^T$ is the weight vector of $a_\varpi(\varpi=1,2,\dots,m)$, satisfying $\omega_i\in[0,1]$ and $\sum_{i=1}^{m}\omega_i=1$, and let $P=(p_1,p_2,\dots,p_m)\in R^m$ be a vector of parameters. If

$$Cq-ROFWDMM^{P}\,(a_1,a_2,\dots,a_m)=\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}\left(\prod_{\vartheta\in S_m}\sum_{\varpi=1}^{m}\left(P_\varpi a_{\vartheta(\varpi)}^{m\omega_{\vartheta(\varpi)}}\right)\right)^{\frac{1}{\varpi!}}$$

Subsequently $Cq-ROFWDMM^{P}$ is the Cq-ROAWDMM, where $\vartheta\,(\varpi)\,(\varpi=1,2,3,\dots,m)$ is the permutation of $(1,2,3,\dots,m)$, and S_m is the group of all permutations of $(1,2,3,\dots,m)$.

Thus, the theorem is simply constructed.

Theorem 8: Let $a_\varpi=\left(u_\varpi e^{2i\pi(\phi_\varpi)},v_\varpi e^{2i\pi(\psi_\varpi)}\right)(\varpi=1,2,3,\dots,m)$ be a collection of Cq-ROFVs and $P=(P_1,P_2,\dots,P_m)\in R^m$ is a vector of parameters. Thus, the total value obtained by applying the Cq-ROFWDMM operator remains a Cq-ROFV.

and

$$Cq-ROFWDMM^{p}(a_1,a_2,\dots,a_m)=\left(\begin{array}{c}\left(1-\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-u_{\vartheta(\varpi)}^{qm\omega}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{1/q} e^{2\pi i\left(\left(1-\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\phi_{\vartheta(\varpi)}^{qm\omega}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{\sum_{\varpi=1}^{m}P_\varpi}}\right)^{1/q}\right)},\\ \left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\left(1-v_{\vartheta(\varpi)}^{q}\right)^{m\omega}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{q\sum_{\varpi=1}^{m}P_\varpi}} e^{2\pi i\left(\left(1-\prod_{\vartheta\in S_m}\left(1-\prod_{\varpi=1}^{m}\left(1-\left(1-\psi_{\vartheta(\varpi)}^{q}\right)^{m\omega}\right)^{P_\varpi}\right)^{\frac{1}{m!}}\right)^{\frac{1}{q\sum_{\varpi=1}^{m}P_\varpi}}\right)}\end{array}\right)$$

5. Overview of Intelligent Decision Algorithm

In this section, the Cq-ROFSs are useful instruments for characterizing the fuzziness, uncertainty, and indeterminacy of decision makers. They are therefore frequently utilized in MADM difficulties.

Assume that there are n alternatives $A=(A_1,A_2,\dots,A_n)$, collection of m attributes $X=(X_1,X_2,\dots,X_m)$ with weight vectors, $W=(w_1,w_2,\dots,w_m)^T$, where $\sum_{\varpi=1}^{m}w_i=1$. Also assume that $R=(r_{i\varpi})_{n\times m}=\left(u_{i\varpi}e^{i2\pi(\phi_{i\varpi})},v_{i\varpi}e^{i2\pi(\psi_{i\varpi})}\right)$ is the Cq-ROF matrix. It is composed of Cq-ROFVs, with each pair consisting of complex-valued MD and complex-valued NMD. Here, $u_{i\varpi}e^{i2\pi(\phi_{i\varpi})}$ denotes the degree to which the alternatives A_i satisfy the attributes X_i provided by the decision maker, and $v_{i\varpi}e^{i2\pi(\psi_{i\varpi})}$ denotes the degree to which the alternatives do not satisfy the attributes X_i provided by the decision makers.

An algorithm for the MADM problem is used to carry out the evaluation and aggregation phase. The following are the steps of this procedure.

Step 1: A decision matrix serves as a visual representation of the alternatives and qualities that the decision-maker has gathered from the Cq-ROFS.

$$R=(r_{ij})_{n\times m}=\begin{bmatrix}\left(u_{11}e^{i2\pi(\phi_{11})},v_{11}e^{i2\pi(\psi_{11})}\right) & \left(u_{12}e^{i2\pi(\phi_{12})},v_{12}e^{i2\pi(\psi_{12})}\right) & \cdots\left(u_{1m}e^{i2\pi(\phi_{1m})},v_{1m}e^{i2\pi(\psi_{1m})}\right)\\ \left(u_{21}e^{i2\pi(\phi_{21})},v_{21}e^{i2\pi(\psi_{21})}\right) & \left(u_{22}e^{i2\pi(\phi_{22})},v_{22}e^{i2\pi(\psi_{22})}\right) & \dots\left(u_{2m}e^{i2\pi(\phi_{2m})},v_{2m}e^{i2\pi(\psi_{2m})}\right)\\ \vdots & \vdots & \vdots\\ \left(u_{n1}e^{i2\pi(\phi_{n1})},v_{n1}e^{i2\pi(\psi_{n1})}\right) & \left(u_{n2}e^{i2\pi(\phi_{n2})},v_{n2}e^{i2\pi(\psi_{n2})}\right) & \dots\left(u_{nm}e^{i2\pi(\phi_{nm})},v_{nm}e^{i2\pi(\psi_{nm})}\right)\end{bmatrix}$$

Step 2: We get the normalized matrix of the decision matrix $R=(r_{i\varpi})_{n\times m}=\left(u_{i\varpi}e^{i2\pi(\phi_{i\varpi})},v_{i\varpi}e^{i2\pi(\psi_{i\varpi})}\right)$.

$$r_{i\varpi}=\begin{cases}R_{i\varpi}^{c} & \varpi\in B\\ R_{i\varpi} & \varpi\in C\end{cases}$$

Step 3: Compute degree of weights for each attribute using the following expression:

$$w_\varpi=\frac{u_{i\varpi}+r_{i\varpi}\left(\frac{u_{i\varpi}}{u_{i\varpi}+v_{i\varpi}}\right)}{\sum_{\varpi=1}^{m}\left(u_{i\varpi}+r_{i\varpi}\left(\frac{u_{i\varpi}}{u_{i\varpi}+v_{i\varpi}}\right)\right)}+\frac{\phi_{i\varpi}+r_{i\varpi}\left(\frac{\phi_{i\varpi}}{\phi_{i\varpi}+\psi_{i\varpi}}\right)}{\sum_{\varpi=1}^{m}\left(\phi_{i\varpi}+r_{i\varpi}\left(\frac{\phi_{i\varpi}}{\phi_{i\varpi}+\psi_{i\varpi}}\right)\right)}$$

Step 3: We explore the global values using the purposed Cq-ROFWMM operator.

$$Cq - ROFWMM^{p}\ (a_{i1}, a_{i2}, \ldots, a_{im}) = a_i$$
$$Cq - ROFWDMM^{p}\ (a_{i1}, a_{i2}, \ldots, a_{im}) = a_i$$

Step 4: In this step, we look into each consequence's score values of step 3 and step 4.

Step 5: After ranking every outcome of the score values, select the attribute that is most appropriate.

5.1 Case Study

MADM is a common practice in the field of medical diagnostics, when a group of experts analyses multiple facets of suspected medical diseases in order to come to a consensus diagnosis. Medical data contains intrinsic ambiguity and fuzziness; which traditional approaches may find difficult to handle. In order to improve DM processes in medical diagnostics, the research study presents a unique method that combines MM operators with Cq-ROFSs. The steps required are as follows:

We assumed the degree of estimated weight for each characteristic in this experimental case study in order to assess the best option. Thus, we can also use various approaches or strategies to look at the weight of the criteria. We are going to study X-ray machine of different companies taking as alternatives $(A_1, A_2, A_3, A_4, A_5)$ on the basis of some features or attributes of X-ray machine under MADM method by using Cq-ROFWMM and Cq-ROFWDMM operators. The attributes are as follows.

Image Quality C_1: Detailed diagnosis through high-resolution imaging.

Radiation Dose C_2: The amount of radiation given to the patient, which is commonly expressed in milligrams, is important to consider when attempting to balance patient safety and picture quality. Higher doses can enhance image detail, while lower doses lower risk.

Exposure Time C_3: The amount of time the X-ray tube is in operation. This affects both the image quality and the overall radiation dose given to the patient.

Integration with IT Systems C_4: For effective data management, a seamless connection with healthcare networks is necessary.

5.2 Evaluation Process

Step 1: A decision matrix of Table 1 serves as a visual representation of the alternatives and qualities that the decision-maker has gathered from the Cq-ROF.

Table 1
Cq-ROF data is arranged in a decision matrix

Alternatives	C_1	C_2
A_1	$\left(0.55e^{2\pi i(0.63)}, 0.88e^{2\pi i(0.65)}\right)$	$\left(0.65e^{2\pi i(0..80)}, 0.45e^{2\pi i(0.62)}\right)$
A_2	$\left(0.76e^{2\pi i(0.38)}, 0.65e^{2\pi i(0.71)}\right)$	$\left(0.76e^{2\pi i(0.87)}, 0.41e^{2\pi i(0.37)}\right)$
A_3	$\left(0.74e^{2\pi i(0.61)}, 0.45e^{2\pi i(0.83)}\right)$	$\left(0.77e^{2\pi i(0.66)}, 0.26e^{2\pi i(0.55)}\right)$
A_4	$\left(0.83e^{2\pi i(0.49)}, 0.34e^{2\pi i(0.54)}\right)$	$\left(0.68e^{2\pi i(0.69)}, 0.53e^{2\pi i(0.35)}\right)$
A_5	$\left(0.65e^{2\pi i(0.47)}, 0.58e^{2\pi i(0.88)}\right)$	$\left(0.64e^{2\pi i(0.58)}, 0.46e^{2\pi i(0.48)}\right)$
	C_3	C_4
A_1	$\left(0.73e^{2\pi i(0.45)}, 0.51e^{2\pi i(0.75)}\right)$	$\left(0.44e^{2\pi i(0.75)}, 0.71e^{2\pi i(0.53)}\right)$
A_2	$\left(0.65e^{2\pi i(0.59)}, 0.76e^{2\pi i(0.68)}\right)$	$\left(0.62e^{2\pi i(0.78)}, 0.52e^{2\pi i(0.54)}\right)$
A_3	$\left(0.85e^{2\pi i(0.87)}, 0.41e^{2\pi i(0.58)}\right)$	$\left(0.48e^{2\pi i(0.88)}, 0.56e^{2\pi i(0.45)}\right)$
A_4	$\left(0.62e^{2\pi i(0.68)}, 0.72e^{2\pi i(0.83)}\right)$	$\left(0.53e^{2\pi i(0.75)}, 0.38e^{2\pi i(0.54)}\right)$
A_5	$\left(0.82e^{2\pi i(0.79)}, 0.68e^{2\pi i(0.48)}\right)$	$\left(0.46e^{2\pi i(0.53)}, 0.88e^{2\pi i(0.76)}\right)$

Step 2: Due to the fact that Table 1 only contains one type of helpful information, normalizing the supplied decision matrix into a normalized decision matrix is not necessary.

Step 3: Estimated weights based on step 3 are given by:

$$\begin{pmatrix} 0.2222 & 0.2904 & 0.2430 & 0.2444 \\ 0.2081 & 0.3036 & 0.2221 & 0.2663 \\ 0.2264 & 0.2682 & 0.2700 & 0.2354 \\ 0.2582 & 0.2730 & 0.2116 & 0.2571 \\ 0.2248 & 0.2824 & 0.3004 & 0.1924 \end{pmatrix}$$

Step 4: Table 2 displays aggregate data of various preferences or attributes relating to each choice and computed outcomes using derived mathematical approaches of the Cq-ROFWMM and Cq-ROFWDMM operators at a fixed value.

Table 2
Results aggregated by the Cq-ROFWMM and Cq-ROFWDMM operators

Alternatives	Cq-ROFWMM	Cq-ROFWDMM
A_1	$\left(0.5812e^{2\pi i(0.6412)}, 0.7143e^{2\pi i(0.6575)}\right)$	$\left(0.6307e^{2\pi i(0.7031)}, 0.6144e^{2\pi i(0.6317)}\right)$
A_2	$\left(0.6921e^{2\pi i(0.6224)}, 0.6333e^{2\pi i(0.6192)}\right)$	$\left(0.7138e^{2\pi i(0.7414)}, 0.5676e^{2\pi i(0.5553)}\right)$
A_3	$\left(0.6939e^{2\pi i(0.7444)}, 0.4576e^{2\pi i(0.6576)}\right)$	$\left(0.7537e^{2\pi i(0.8014)}, 0.4044e^{2\pi i(0.5870)}\right)$
A_4	$\left(0.6553e^{2\pi i(0.6435)}, 0.5572e^{2\pi i(0.6475)}\right)$	$\left(0.6952e^{2\pi i(0.6803)}, 0.4705e^{2\pi i(0.5386)}\right)$
A_5	$\left(0.6264e^{2\pi i(0.5784)}, 0.7251e^{2\pi i(0.7318)}\right)$	$\left(0.6939e^{2\pi i(0.6492)}, 0.6288e^{2\pi i(0.6234)}\right)$

Step 5: Compute each alternative score values by applying definition. The score values for each alternative or individual are displayed in Table 3.

Table 3
Computed score values of all alternatives.

	A_1	A_2	A_3	A_4	A_5	Ordering of preferences
Cq-RODWMM	0.2028	0.2703	0.3416	0.2758	0.1665	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
Cq-RODWDMM	0.2786	0.3543	0.4186	0.3476	0.2792	$A_3 \succ A_2 \succ A_3 \succ A_5 \succ A_1$

Step 6: The ranking and ordering of all score values related to each alternative are also controlled in Table 3, which allows you to look into the best option for a particular alternative. Additionally, Figure 1 provides the reader with a clearer picture of the combined consequences and results of every option. symmetrical representations play a major role in the process of decision analysis and aggregation, and Figure 1 various colors correspond to the dependability of various options.

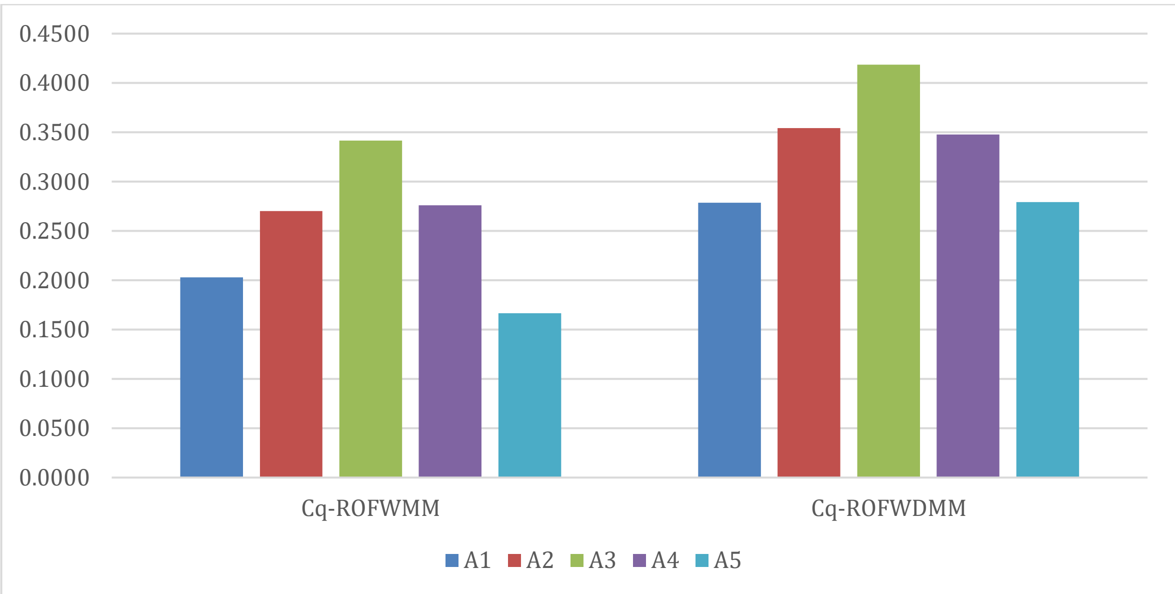

Fig. 1. Results of score values by Cq-ROFWMM and Cq-ROFWDMM operators

5.3 Sensitive Study

In this subsection, the authors use an algorithm for the MADM problem to show the applicability and resilience of the derived techniques. In the process of DM, decision-makers combine the scores and the related ranking of the options by using the various values n. This technique's distinctive quality is its ability to demonstrate consistency and intensity during the DM process. We now show the aggregated results of the MADM problem at various values of n, as produced by the Cq-ROFWMM and Cq-ROFWDMM operators. Table 3 displays the combined results of the different ranking and score values. Upon examination of the calculated score values, it is evident that alternative A_3 is greator at various levels then others. Table 3 displays the combined results of the Cq-ROFWMM and Cq-ROFWDMM operators. Another element of the MADM problem that offers a clear comprehension of the decision support system is geometric shape.

The impact of the parametric vector P on the overall assessment evaluation function and the final ranking outcomes are shown in Tables 4 and 5.

Table 4
Performance ranking of different parametric vectors by using the Cq-ROFWMM operator

Parametric vector	Evaluation function $\delta(A_i), (i = 1,2,3,4,5)$	Ranking results
(2,1,1,1)	$\delta(A_1) = 0.1850, \delta(A_2) = 0.2541, \delta(A_3) = 0.3192, \delta(A_4) = 0.2884, \delta(A_5) = 0.1553$	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
(1,1,1,1)	$\delta(A_1) = 0.2028, \delta(A_2) = 0.2703, \delta(A_3) = 0.3416, \delta(A_4) = 0.2758, \delta(A_5) = 0.1665$	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
(1,1,0,0)	$\delta(A_1) = 0.2161, \delta(A_2) = 0.3105, \delta(A_3) = 0.3089, \delta(A_4) = 0.3537, \delta(A_5) = 0.1986$	$A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$
(1,0,0,0)	$\delta(A_1) = 0.1321, \delta(A_2) = 0.2356, \delta(A_3) = 0.2562, \delta(A_4) = 0.3844, \delta(A_5) = 0.1398$	$A_4 \succ A_3 \succ A_2 \succ A_5 \succ A_1$
(1,1,1,0)	$\delta(A_1) = 0.2002, \delta(A_2) = 0.2555, \delta(A_3) = 0.3573, \delta(A_4) = 0.2668, \delta(A_5) = 0.2396$	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_1$
(1,0.50,0.50,0.50)	$\delta(A_1) = 0.1815, \delta(A_2) = 0.2519, \delta(A_3) = 0.3168, \delta(A_4) = 0.2871, \delta(A_5) = 0.1519$	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
(1,0.50,0.25,0)	$\delta(A_1) = 0.1751, \delta(A_2) = 0.2542, \delta(A_3) = 0.3084, \delta(A_4) = 0.3147, \delta(A_5) = 0.1888$	$A_4 \succ A_3 \succ A_2 \succ A_5 \succ A_1$
(1,0.25,0.25,0)	$\delta(A_1) = 0.1535, \delta(A_2) = 0.2319, \delta(A_3) = 0.2979, \delta(A_4) = 0.3112, \delta(A_5) = 0.1753$	$A_4 \succ A_3 \succ A_2 \succ A_5 \succ A_1$
(0.50,0.25,0.25,0.25)	$\delta(A_1) = 0.1798, \delta(A_2) = 0.2506, \delta(A_3) = 0.3156, \delta(A_4) = 0.2862, \delta(A_5) = 0.1502$	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
(0.50,0.25,0.25,0)	$\delta(A_1) = 0.1718, \delta(A_2) = 0.2390, \delta(A_3) = 0.3243, \delta(A_4) = 0.2854, \delta(A_5) = 0.2018$	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
(0.25,0.25,0.25,0.25)	$\delta(A_1) = 0.1995, \delta(A_2) = 0.2676, \delta(A_3) = 0.3393, \delta(A_4) = 0.2734, \delta(A_5) = 0.1633$	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
(1,0.25,0.25,0.25)	$\delta(A_1) = 0.1616, \delta(A_2) = 0.2383, \delta(A_3) = 0.2933, \delta(A_4) = 0.3054, \delta(A_5) = 0.1408$	$A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$

Table 5
Performance ranking of distinct parametric vectors by using the Cq-ROFWDMM operator

Parametric vector	Evaluation function $\delta(A_i), (i = 1,2,3,4,5)$	Ranking results
(2,1,1,1)	$\delta(A_1) = 0.2549, \delta(A_2) = 0.3316, \delta(A_3) = 0.3910, \delta(A_4) = 0.3513, \delta(A_5) = 0.2582$	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_1$
(1,1,1,1)	$\delta(A_1) = 0.2786, \delta(A_2) = 0.3543, \delta(A_3) = 0.4186, \delta(A_4) = 0.3476, \delta(A_5) = 0.2792$	$A_3 \succ A_2 \succ A_5 \succ A_5 \succ A_1$
(1,1,0,0)	$\delta(A_1) = 0.2854, \delta(A_2) = 0.4056, \delta(A_3) = 0.3397, \delta(A_4) = 0.3835, \delta(A_5) = 0.2608$	$A_2 \succ A_4 \succ A_3 \succ A_1 \succ A_5$

Table 5
Continued

Parametric vector	Evaluation function $\delta(A_i), (i = 1,2,3,4,5)$	Ranking results
$(1,0,0,0)$	$\delta(A_1) = 0.1046, \delta(A_2) = 0.2050, \delta(A_3) = 0.2287,$ $\delta(A_4) = 0.3577, \delta(A_5) = 0.1162$	$A_4 \succ A_3 \succ A_2 \succ A_5 \succ A_1$
$(1,1,1,0)$	$\delta(A_1) = 0.2786, \delta(A_2) = 0.3501, \delta(A_3) = 0.4085,$ $\delta(A_4) = 0.3433, \delta(A_5) = 0.3306$	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$
$(1,0.50,0.50,0.50)$	$\delta(A_1) = 0.2580, \delta(A_2) = 0.3357, \delta(A_3) = 0.3948,$ $\delta(A_4) = 0.3573, \delta(A_5) = 0.2614$	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_1$
$(1,0.50,0.25,0)$	$\delta(A_1) = 0.2508, \delta(A_2) = 0.3403, \delta(A_3) = 0.3516,$ $\delta(A_4) = 0.3685, \delta(A_5) = 0.2714$	$A_4 \succ A_3 \succ A_2 \succ A_5 \succ A_1$
$(1,0.25,0.25,0)$	$\delta(A_1) = 0.2204, \delta(A_2) = 0.2952, \delta(A_3) = 0.3390,$ $\delta(A_4) = 0.3627, \delta(A_5) = 0.2552$	$A_4 \succ A_3 \succ A_2 \succ A_5 \succ A_1$
$(0.50,0.25,0.25,0.25)$	$\delta(A_1) = 0.2607, \delta(A_2) = 0.3388, \delta(A_3) = 0.3980,$ $\delta(A_4) = 0.3611, \delta(A_5) = 0.2643$	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_1$
$(0.50,0.25,0.25,0)$	$\delta(A_1) = 0.2569, \delta(A_2) = 0.3294, \delta(A_3) = 0.3805,$ $\delta(A_4) = 0.3598, \delta(A_5) = 0.3010$	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_1$
$(0.25,0.25,0.25,0.25)$	$\delta(A_1) = 0.2839, \delta(A_2) = 0.3601, \delta(A_3) = 0.4242,$ $\delta(A_4) = 0.3539, \delta(A_5) = 0.2847$	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ 1$
$(1,0.25,0.25,0.25)$	$\delta(A_1) = 0.2282, \delta(A_2) = 0.3089, \delta(A_3) = 0.3612,$ $\delta(A_4) = 0.3640, \delta(A_5) = 0.2345$	$A_4 \succ A_3 \succ A_2 \succ A_5 \succ A_1$

6. Comparison with Existing Methods

In this section, we provide a comparative analysis of our proposed work of Cq-ROFWMM and Cq-ROFWDMM operators with previously existing methods. In order to do this, we researched numerous existing realistic mathematical techniques as well as original theories. Zhao [45] developed a Hamy mean operator (HMO) based on bipolar complex fuzzy theory but failed to handle the DM problems. Hussain *et al.,* [46] approach to MADM for vendor management analysis with the use of HMO and modern Complex Picture Fuzzy also fails. On the other hand, Garg [47] worked on HMO based on C-ROF setting and their application in MADM and Cq-ROFS. Du *et al.,* [48] developed mathematical approaches to Cq-ROF Frank AOs and used them for DM with several attributes. Yager [49] examined a number of AOs that consider geometric and arithmetic mean operators theories. Javeed *et al.,* [50] presented a study employing the AOs in the Cq-ROF environment. We also compared the results of the approach with the operators proposed by Ali [42] and Liu [51]. The comparative results ranking of all these operators are shown in Table 6. Figure 2 and Figure 3 also shows a geometrical representation to highlight the benefits of our suggested work.

Table 6
Comparative assessments of our proposed work with several existing methods

Environments	Ranking of alternatives
Cq-ROFWMM (Proposed work)	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
Cq-ROFWDMM (Proposed work)	$A_3 \succ A_2 \succ A_3 \succ A_5 \succ A_1$
Cq-ROFFWA [48]	$A_3 \succ A_1 \succ A_5 \succ A_4 \succ A_2$
Cq-ROFFWG [48]	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_1$
Cq-ROFYWA [50]	$A_1 \succ A_3 \succ A_4 \succ A_5 \succ A_2$
Cq-ROFFWG [50]	$A_3 \succ A_5 \succ A_1 \succ A_4 \succ A_2$
Cq-ROFWA [51]	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
Cq-ROFWG [51]	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
Cq-ROFDWA [42]	$A_3 \succ A_1 \succ A_5 \succ A_4 \succ A_2$
Cq-ROFDWG [42]	$A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5$
Cq-ROFWHM [47]	$A_3 \succ A_5 \succ A_1 \succ A_4 \succ A_2$
Cq-ROFWDHM [47]	$A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$
Zhao *et al.,* [45]	Fail
Hussain *et al.,* [46]	Fail

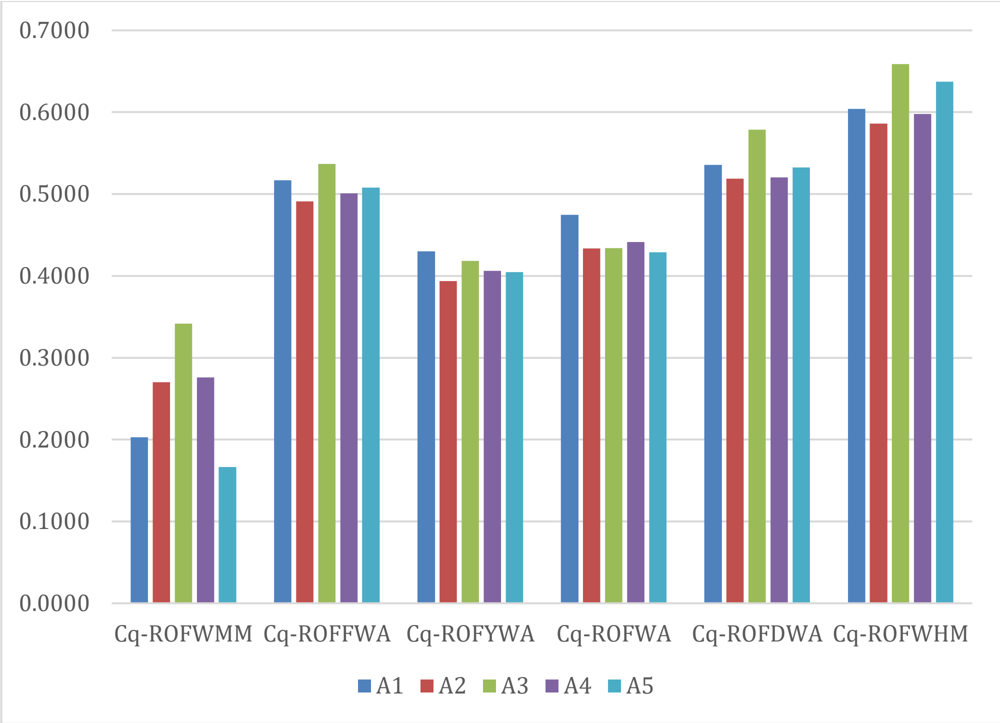

Fig. 2. Comparison with previous AOs

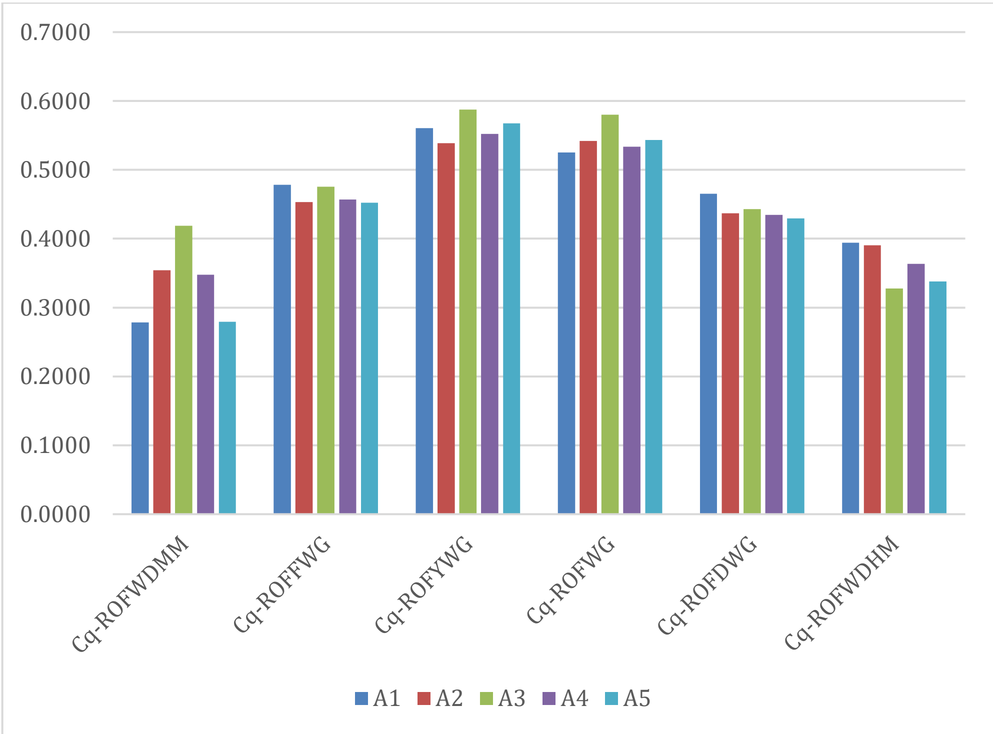

Fig. 3. Comparison with previous geometric operators

To sum up, our approaches are more powerful, helpful, and reasonable than existing methods since they are based on Cq-ROFNs and can take into account the interaction among all aggregated inputs.

7. Conclusion

A novel decision technique for the MADM problem is demonstrated to address challenging real-world applications and various case studies using experimentation. In medical diagnostics, the integration of MM operators with Cq-ROFS offers a potent tool for MADM. This method improves the capacity to manage ambiguity and complexity, resulting in more precise and trustworthy medical diagnoses. The case study demonstrates the usefulness and practical use of these cutting-edge fuzzy techniques in enhancing medical DM procedures through Cq-ROFS and the idea of the MM method. We created a range of practical mathematical techniques based on data, such as Cq-ROFMM, Cq-ROFDMM, Cq-ROFWMM and Cq-ROFWDMM operators. To demonstrate the reliability and validity of the developed techniques, a few noteworthy traits and qualities are also discussed. Furthermore, an algorithm is developed to tackle complex real-world applications of the MADM issue by taking into derived mathematical techniques and research projects. To assess, a numerical example is examined. A perfect answer to the available options and demonstrates the consistency of the obtained mathematical techniques. Additionally, examined are the benefits and coherence of invented research projects. In conclusion, we highlighted the superiority and efficacy of the diagnosed research project using an approach for comparing it to previous research projects.

We observed that while these mathematical techniques have numerous benefits, they are not without restrictions and disadvantages.

Strengths:

i. To use the intricate Cq-ROF with Muirhead procedures for medical diagnosis.
ii. To show how well these techniques work with complicated, ambiguous, and fuzzy data.
iii. To increase the precision and dependability of collective DM in the diagnosis of medical issues.
iv. Provides a strong and adaptable framework for scenarios involving complicated DM.

Limitations:

i. The precise identification and representation of appropriate characteristics are critical to the method's efficacy.
ii. The method's complexity may require immense computing investment.

Further studies may concentrate on extending the use of this approach by including additional healthcare domains, investigating how it integrates with other complex computational methodologies, and executing more extensive trials to bolster its effectiveness, we will use the suggested approach to solve real-world MADM issues.

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Conflicts of Interest

The authors declare no conflicts of interest.

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