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Finding Most Important Criteria in Women's Empowerment for Sports Sector by Pentagonal Fuzzy DEMATEL Methodology

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ABSTRACT

Research on women's empowerment in the sports sector is a key factor in considering the important issues that impact women's leadership, participation, and achievement in sports. Recognizing various key challenges in research helps to develop targeted and suitable strategies and favorable policies that promote equity and inclusivity. In this research article, the approaches for finding the most important criteria in women's empowerment in sports are demonstrated. The relevant criteria are shortlisted by conducting a literature review, collecting experts' opinions, and engaging in direct interaction with associated persons in the sports sector. The solution methodology for finding the criteria preferences is taken as a well-known multi-criterion decision-making method, namely the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method. Due to the presence of uncertainty in the data sets, pentagonal fuzzy set data are considered. The whole proposed approach is followed by numerical and graphical illustrations.

1. Introduction

To create an equitable and equal society, women must be empowered by providing them more tools, chances and support to achieve their full potential and talent. It promotes independence, equality, and the ability to make informed their own choices and decisions. Sports is such a field that helps women's empowerment in a greater way. So, finding important criteria can play the main role in women's empowerment in sports. This work is actually structured based on this motivation. In this

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paper, the fuzzy set and number are deemed as a mathematical tool. In particular, Pentagonal Fuzzy Numbers (PFN) are used to solve this problem. Multi criteria decision making (MCDM) can be chosen as an application for searching the most important criteria for women's empowerment in the sports sector. Here, we choose the Decision Making Trial and Evaluation Laboratory (DEMATEL) methodology in MCDM to process this research work.

1.1 Women empowerment through Sports

Sports inspire women in such a ways that go beyond simple physical exercise; it encourage them to develop resilience, leadership and self-confidence and make them to be an expert in every field. Women who are connected with sports can accept different challenges of social standards and conventional gender roles that are called gender equality while developing their own life skills that benefit them in various aspects, including personal relationships and career development. Each and every woman can bring about social change; they may express their abilities, create women's networks, fight for equality and encourage the upcoming generations. As well as encouraging women to take up leadership roles in sports, it helps them to gain glory and participate more actively in their communities in the wider society. This women's empowerment actually goes beyond the playing field and spreads to larger areas.

1.2 Fuzzy set for modelling real-world problems

Fuzzy sets provide more flexibility in quantifying and reaching an exact solution by evaluating uncertain concepts. This was proposed by Lotfi A. Zadeh in 1965 [1]. Fuzzy sets are very helpful in fields like decision-making and machine learning because of it's flexibility. Moreover, various industries like robotics, consumer electronics, industrial automation, power engineering, image processing, control systems engineering and optimisation have achieved success with fuzzy logic in vague environments. The following are some essential works on uncertain sets and numbers that can provide initial ideas, theories and frameworks, i.e., [2-5]. Here are some of the real world applications of fuzzy sets, they are [6-9].

In this paper, we apply the Pentagonal Fuzzy Set (PFS) to solve this research work to find the most important criteria in women empowerment for sports sector. We indicate here some papers of real world problems that contain Pentagonal Fuzzy Number (PFN), i.e., fuzzy mellin transformation [10], selection of cloud service providers [11], games problem application [12], initial properties of PFN and application in fuzzy equation [13], etc.

1.3 MCDM methods for decision making

Multi-Criteria Decision-Making (MCDM) methodology [6] is an optimization process or decision making process to evaluate and rank the alternatives considering various conflicting criteria and sub-criteria. This method is useful for dealing with complicated situations where solutions encompass several variables which may be quantitative, qualitative or sometimes both. Several MCDM methods are employed in the decision-making process, some are Analytic Hierarchy Process (AHP) [7], analytic network process (ANP) [14], Entropy [6], CRiteria Importance Through Intercriteria Correlation (CRITIC) [15], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [16], Complex Proportional Assessment (COPRAS) [9], Weighted Aggregates Sum Product Assessment (WASPAS) [8], Combined Compromise Solution (CoCoSo) [8], Multi-Objective Optimization Ratio Analysis (MOORA) [15], Vlekriterijumsko KOmpromisno Rangiranje (VIKOR) [6], Decision making trial and evaluation laboratory (DEMATEL) [17], and so on.

The DEMATEL based MCDM method [17] is one of the popular optimization techniques used to utilised to analyze and display the connections and impacts among a group of criteria and sub-criteria. It is particularly effective well when dealing with complex systems with interconnected criteria. The DEMATEL method is applied in numerous fields, including education fields by Tzeng, G. et al. [18], supply chain risk management by Samvedi, A. et al. [19], airlines traffic controller systems by Bongo, M. F. et al. [20], auditing & risk control management by Tsai, W. et al. [21]. Furthermore, the DEMATEL methodology was also applied in hospital service management by Shieh, J. et al. [22], quality model of cost quality by Tsai, W. et al. [14], split-half coefficients analysis by Shieh, J. et al. [23], industrial fields by Lin, Y. et al. [24]. Additionally, in construction sites [25], knowledge management application fields [26], an outreach program for employment services [27], emergency management [28] and so many fields DEMATEL techniques evaluated the results.

1.4 Structure of the paper

This section discusses the structure of the research. The introduction of this study is described in Section 1. Then, the criteria for women's empowerment in the sports sector are covered in Section 2. Additionally, the fuzzy set and its pentagonal extensions are discussed in Section 3 and the fuzzy DEMATEL method is described in Section 4, respectively. The model formulation and data collection are performed in Section 5. Further, numerical illustration and discussion are conducted in Section 6. In conclusion, the conclusion and future research scope are described in Section 7.

2. Criteria for women empowerment in sports sector

It wasn't until the early 20th century the women and sports even appeared in the same sentence. Until then the idea of women competing in the sports and paid was unacceptable to the then patriarchal society. Empowering women through sports [29–31] is very much required to enhance and/or address various socio-economical factors like establishment of professional network, develop sense of identity, promote gender-equality, women's leadership roles at society, participation in socio-cultural decision-making etc. In compare to other groups, women are more inclined to lead sedentary lives and have higher rates of related illnesses [32]. It is noted that sport activities of women helps them to boost their confidence, tenacity, self-determination and team-work skills, which helps women into their daily lives and empowered them in many ways. There are various criteria involved in the sports sector that can endeavour to produce a more equitable and empowered surrounding for women, which allowing them to prosper and excel on and off the field. The important criteria for women's empowerment in the sports sector have been presented in pictorial form in Figure 1 and described as follows:

2.1 Supportive Infrastructure (S_1):

Supportive Infrastructure for women's sports is a very crucial issue. The importance of sports infrastructure for participation in public sports has been emphasized by numerous researchers [33]. Being submissive in a highly competitive male-dominated workplace, institutional racism, gender discrimination, marginalisation based on ethnicity & culture, a lack of appropriate resources are some of the barriers to explore sports among women[34]. Supportive Infrastructure for women's sports includes many things like:

- (a) Safe, well-maintained and gender-specific sports facilities.
- (b) Adequate Equipment and gear: Exploring women sport need to access high-quality and properly

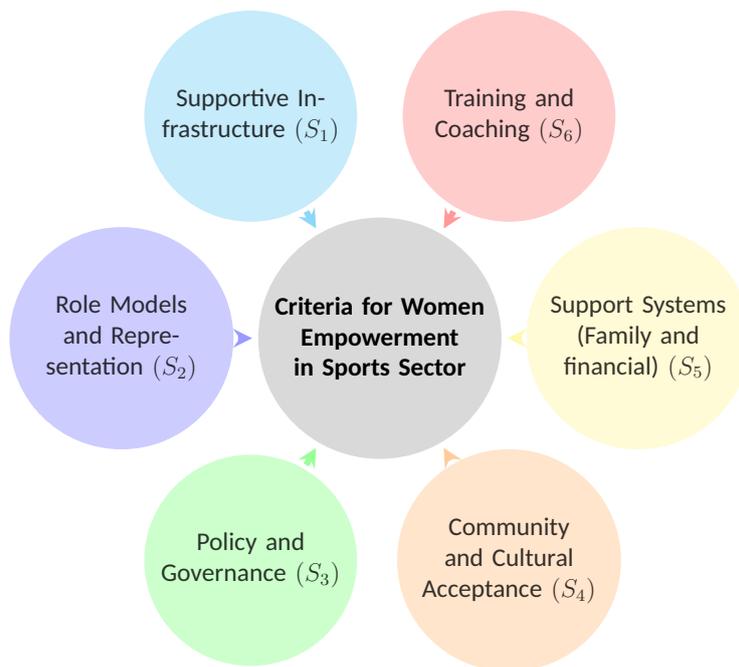


Fig. 1. Different criteria for women empowerment through sports

fitting equipment and gear.

- (c) Scholarships and funding: Financial support for education, training, and competition.
- (d) Medical support: Trained medical professionals and access to specialized medical services for women sportsmen is very much essential.
- (e) Mentorship programs and networking opportunities: Different mentorship programs for women sports person will help to develop a good network with experienced athletes, coaches, mentors and industry professionals.
- (f) Media coverage and promotion: Equal media coverage and promotion of women's sports is needed that is as of men sports.
- (g) Accessibility for women with special challenges: Facilities and equipment adapted for women with disabilities is also needed.
- (h) Transportation and logistics Support : For safety issues and other common problem faced by women, good transportation facilities and other logistic support are needed for women sports person.
- (i) Wellness and Mental health support: Access to sports nutritionists and wellness programs is crucial to boost up women sports. Also trained professionals are needed for mental health and well being of women sports persons.

Access to appropriate facilities, appropriate training and other supportive measures encourages women in sports participation and fosters their talent development. The research data indicates that increased level of physical activity is correlated with the proximity and density of sports facilities [35, 36].

2.2 *Role Models and Representation* (S_2):

Prominent female role models in sports inspire and motivate other women and girls to participate and excel in this field. Representation in media and leadership positions also plays a significant role in empowering women in sports. Women who have participated in athletics exhibit a great love of the game and a drive to assume leadership positions. To highlight the socio-cultural and structural problems that result from the intersection of gender and Indigenouness, further research on Indigenouness women in sports leadership is required [34]. Till date, the coaching profession is dominated by men due to dearth of female in coaching roles [37]. According to [38], female athletes who received coaching from a female head coach were four times more likely to go on to become coaches themselves than those who did not receive such exposure. The significance of female athletes having access to role models who not only share their gender but also participate in sports was emphasized by [39]. Studies have indicated that being around female athletes can lessen discrimination against them and foster a greater sense of familiarity and interest in them [40]. Consequently, encouraging young athletes to interact with women in coaching positions may help to progressively shift attitudes and debunk the myth that coaching ability is gendered. Thus, it aids in the eradication of ingrained prejudices and unintentional connections [41]. Role models and representation play a crucial role in women's sports, as they categorically address and improve the following issues:

- (a) Inspire, influence and motivate young girls and women to participate in sports.
- (b) Seize up gender stereotypes and change the societal norms.
- (c) Showcase and demonstrate women's athletic abilities and accomplishments.
- (d) Demonstrate leadership quality and foster a sense of community and support.
- (e) Encourage diversity and inclusion.
- (f) Provide opportunities for mentorship and guidance.
- (g) Celebrate women's successes and achievements.

2.3 *Policy and Governance* (S_3):

Implementing policies that promote gender equality in sports governance, such as equal pay, anti-discrimination measures, and inclusive leadership, creates an environment where women can thrive and feel valued is very much essential. Inclusive government policies are needed to prevent women harassment and discrimination as well as access to workshops, clinics, educational materials and competition logistics. Also special policies are needed for mental health and well being of women sportsmen. Moreover, the concept of "diversity management" in sports, is necessary to examine the combined effects of gender equality policies and diversity management on indigenous women [34]. Compared to male athletes and men's sports, female athletes and women's sports continue to receive entirely distinct treatment from the sports media commercial complex [42]. The results of research indicate that engagement in sports increases with socioeconomic status. Additionally, there is a correlation between increased sports activity and academic level. Sports are comparatively more expensive and therefore, poverty is one of the beerier in women's sports. Research findings indicate that between 2005 and 2019, there was a rise in income inequality in sports engagement, particularly among women and the elderly [43].

2.4 *Community and Cultural Acceptance* (S_4):

Changing societal attitudes towards women in sports can break down barriers and stereotypes, encouraging more women to participate without fear of judgment or discrimination. Researchers and policy-makers have been particularly interested in the differences of daily mobility patterns among men and women [44]. Social norms are rules that characterize the expected behavior of members of a certain group or community. Social norms that pertain exclusively to the roles that men and women play can be categorized as a subset of gender norms. Men's and women's (often unequal) access to resources and freedoms can be shaped by gender norms, which are "embedded in formal and informal institutions, produced and reproduced through social interaction" [45]. The continuation or discontinuation of women's sports is influenced by the culture of the sports environment. To encourage women to participate in sports, sports facilities must foster an comprehensive and supportive culture [46].

2.5 *Support Systems (Family and financial)* (S_5):

Establishing support systems including mentorship programs, peer networks, and financial assistance helps women navigate challenges and sustain their involvement in sports. A healthy work-life balance is something that many sports organizations struggle with, which makes things harder for female coaches. The demanding nature of coaching, coupled with its unpredictable schedule and frequent travel, might be a challenge for women seeking to reconcile their personal and familial obligations [47]. Furthermore, women frequently encounter sexist or disparaging remarks from co-workers, athletes, and even parents. This damages their professional reputation and fosters a difficult and unpleasant work atmosphere [47]. The study's findings indicate that individuals who had coached women were happier with female coaches, thought more highly of them as potential coaches, and had a lower association between men and sports [41]. One way to combat the widespread prejudice and discrimination in sports coaching is to provide athletes with more opportunities to engage in meaningful interactions with women in coaching jobs [48]. Since childcare is primary responsibilities of all women in our society, so good access to childcare services and family-friendly facilities are needed to boost-up women sports.

2.6 *Training and Coaching* (S_6):

Providing quality coaching and training opportunities specifically tailored to women's needs can enhance their skills and confidence, leading to greater participation and success in sports. Having appropriate coaches, making use of their sports and social networks, mediating relationships with members of the governing body, adopting a collaborative approach, and challenging ethnic, cultural, and gender expectations can help overcome obstacles like a male-dominated, competitive environment, racism, government and gender discrimination, cultural identities, and lack of support [34]. According to research, mentoring roles are frequently filled through unofficial recruitment channels and mostly depend on contacts, personal networks, and social capital. This hiring policy exacerbates the gender gap by giving preference to men, who are typically viewed as capable trainers [49]. The establishment of support systems in sports organizations that actively pushed women to seek leadership roles was covered by [50]. Furthermore, they emphasize the significance of leadership development initiatives created especially for women, which improve women's visibility and confidence in these roles in addition to imparting knowledge and skills. Of these tactics, putting more women in coaching positions is a particularly good way to influence young athletes. Increasing young people's exposure to female instructors is a critical first step in advancing gender equality in sports. Wider educational and societal

initiatives to confront and alter enduring preconceptions, however, must be implemented in addition to this [41]. Qualified coaches and trainers with experience are very much needed in women's sports.

3. Fuzzy set and fuzzy number

This section discusses the Fuzzy Decision making trial and evaluation laboratory (FDEMATEL) method. Fuzzy sets capture the uncertainty of the system and the DEMATEL method is an MCDM method for recognising the inter dependency among the factors. First introduce the fuzzy set, especially special types of fuzzy set namely the Pentagonal Fuzzy Set (PFS), followed by the DEMATEL method in a fuzzy environment.

3.1 Fuzzy Set

The concept of the fuzzy set was first introduced by Lotfi A. Zadeh [1] in 1965. In contrast to crisp set or binary sets in a classical set theory, where an arbitrary element is either included or excluded from the set, but in a fuzzy set theory, it permits every element for a degree of membership. The membership value of an element lies between $[0, 1]$ and 0 denotes the element does not belong to the set, 1 denotes the element belongs to the set and all intermediate values represent the elements belong to partially and proportionally with the membership values.

Definition 1. Fuzzy Set: [4]

Consider \mathcal{X} be a universal set of discourse. A fuzzy set $\tilde{\mathcal{F}}$ on \mathcal{X} define as

$$\tilde{\mathcal{F}} = \{ \langle \xi, \mu_{\tilde{\mathcal{F}}} \rangle : \xi \in \mathcal{X} \} \tag{1}$$

where $\mu_{\tilde{\mathcal{F}}}$ be the membership function of the set $\tilde{\mathcal{F}}$ define on \mathcal{X} to $[0, 1]$.

Example 1. Let a fuzzy set $\tilde{\mathcal{W}}$ define on the universal set $\mathcal{Y} = \{1, 2, 3, \dots, 8\}$ and define as

$$\tilde{\mathcal{W}} = \{(1, 0.5), (2, 0.6), (3, 0.8), (4, 0.2), (5, 0.9), (6, 0), (7, 0.9), (8, 1)\}$$

Then the membership value ($\mu_{\tilde{\mathcal{W}}}$) of the element 1 is 0.5, 2 is 0.6, 3 is 0.8 and so on. The membership value of the element 6 is 0, that is the element 6 is fully not in the set $\tilde{\mathcal{W}}$; simultaneously, the membership value of the element 8 is 1, that is the element 8 is fully belongs to the set $\tilde{\mathcal{W}}$. Furthermore, any other elements in the set \mathcal{Y} partially belong to the fuzzy set $\tilde{\mathcal{W}}$ based on their degree of membership values.

Fuzzy numbers are specific types of fuzzy sets which restrict some condition on the membership function of fuzzy set over the domain of real number line (\mathbb{R}). Fuzzy number can be defined as

Definition 2. Fuzzy Number: [3]

Assume $\tilde{\mathcal{E}}$ be a fuzzy set defined in the universal set \mathcal{X} . Then the fuzzy set $\tilde{\mathcal{E}}$ is called fuzzy number if the universal set $\mathcal{X} = \mathbb{R}$ and the membership function ($\mu_{\tilde{\mathcal{E}}}$) satisfies the following:

1. $\tilde{\mathcal{E}}$ be a normal fuzzy set; i.e., $\exists \psi \in \mathbb{R}$ such that $\mu_{\tilde{\mathcal{E}}}(\psi) = 1$.
2. Support of $\tilde{\mathcal{E}}$ is bounded; i.e., $\text{Sup}(\tilde{\mathcal{E}}) = \{ \psi : \mu_{\tilde{\mathcal{E}}}(\psi) > 0 \} \subset \mathbb{R}$.
3. The membership function ($\mu_{\tilde{\mathcal{E}}}$) of the fuzzy set ($\tilde{\mathcal{E}}$) is piecewise continuous on \mathbb{R} .

4. $\tilde{\mathcal{E}}$ is a convex fuzzy set; i.e., $\forall \psi, \zeta \in \mathbb{R}$ satisfies

$$\mu_{\tilde{\mathcal{E}}}(\lambda\psi + (1 - \lambda)\zeta) \geq \min \{ \mu_{\tilde{\mathcal{E}}}(\psi), \mu_{\tilde{\mathcal{E}}}(\zeta) \} \tag{2}$$

where $\lambda \in [0, 1]$.

Example 2. Consider universal set $\mathcal{X} = \mathbb{R}$ and fuzzy number $\tilde{\mathcal{V}}$ define as

$$\tilde{\mathcal{V}} = \{ \langle \xi, |\sin(\xi)| \rangle ; \xi \in \mathbb{R} \}$$

In the fuzzy number $\tilde{\mathcal{V}}$, for elements $\xi = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$ membership value is $\mu_{\tilde{\mathcal{V}}}(\xi) = 0$, for elements $\xi = \pm\pi/2, \pm3\pi/2, \pm5\pi/2, \dots$ membership value is $\mu_{\tilde{\mathcal{V}}}(\xi) = 1$ and for any other elements $\xi \in \mathbb{R}$ membership value are lies in between $(0, 1)$. Then the fuzzy number $\tilde{\mathcal{V}}$ satisfies all properties described in Definition 2.

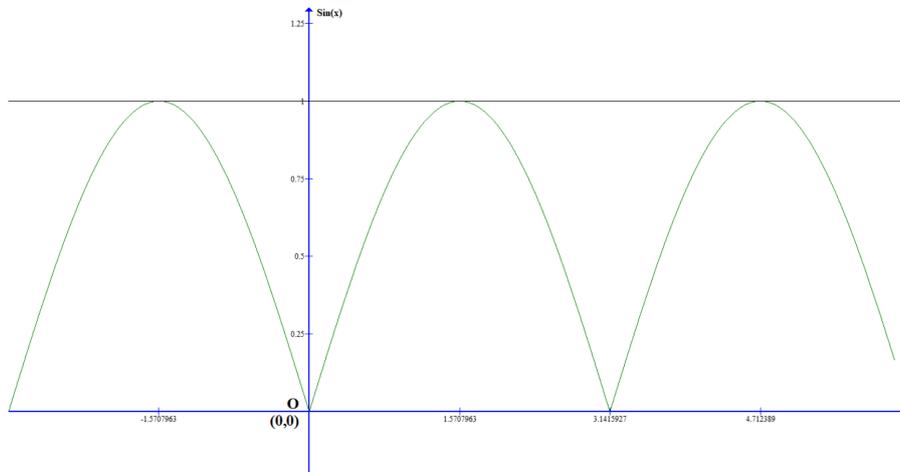


Fig. 2. Graphical structure of the fuzzy set ($\tilde{\mathcal{V}}$)

Remark 1. Fuzzy numbers [3] can handle the uncertainty and imprecision of the system. In the present era, it is applied in numerous fields in mathematics and computer science where precise measurements are not always possible or necessary. Graphical diagram of the fuzzy number described in Example 2 is represented in Figure 2.

3.2 Pentagonal Fuzzy Set (PFS)

A pentagonal fuzzy set [11] represents fuzzy variables in a manner that captures the degree of membership of elements in a set with a shape resembling a trapezoid. There are several extensions of fuzzy sets exist, some of them are triangular fuzzy set (TFS) [51], trapezoidal fuzzy set (TrFS) [52], pentagonal fuzzy set (PFS) [11], hexagonal fuzzy set (HFS) [53], intuitionistic fuzzy set (IFS) [54], neutrosophic fuzzy set (NFS) [9] and so on. The Pentagonal Fuzzy Set (PFS) is defined as follows:

Definition 3. Pentagonal Fuzzy Set: [11]

Consider \mathcal{X} be a universal set. A pentagonal fuzzy set $\tilde{\mathcal{P}}$ define on \mathcal{X} and define as

$$\tilde{\mathcal{P}} = \{ \langle \xi; (\rho_1, \rho_2, \rho_3, \rho_4, \rho_5), \phi \rangle ; \mu_{\tilde{\mathcal{P}}}(\xi) \} \tag{3}$$

where the membership function $\mu_{\tilde{\mathcal{P}}}$ presented as

$$\mu_{\tilde{\mathcal{P}}}(\xi) = \begin{cases} 0 & \text{when } \xi < \rho_1 \\ \phi \frac{\xi - \rho_1}{\rho_2 - \rho_1} & \text{when } \rho_1 \leq \xi < \rho_2 \\ \phi + (1 - \phi) \frac{\xi - \rho_2}{\rho_3 - \rho_2} & \text{when } \rho_2 \leq \xi < \rho_3 \\ 1 & \text{when } \xi = \rho_3 \\ \phi + (1 - \phi) \frac{\rho_4 - \xi}{\rho_4 - \rho_3} & \text{when } \rho_3 < \xi \leq \rho_4 \\ \phi \frac{\rho_5 - \xi}{\rho_5 - \rho_4} & \text{when } \rho_4 < \xi \leq \rho_5 \\ 0 & \text{when } \rho_5 < \xi \end{cases} \quad (4)$$

and ϕ are two extreme points of the pentagon with $0 \leq \phi \leq 1$.

Here, in PFS the values $\rho_1, \rho_2, \rho_3, \rho_4$ and ρ_5 are arbitrary elements in \mathcal{X} with increasing order ($\rho_1 \leq \rho_2 \leq \rho_3 \leq \rho_4 \leq \rho_5$). In the membership function ($\mu_{\tilde{\mathcal{P}}}$) of the pentagonal fuzzy set ($\tilde{\mathcal{P}}$) is a pentagon form define in Equation (4) and graphically represented on Figure 3, respectively.

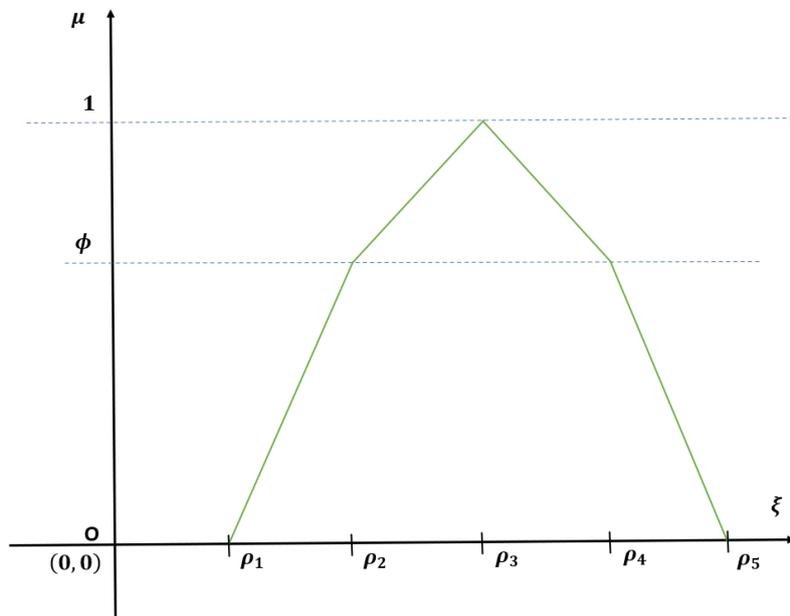


Fig. 3. Structural diagram of the Pentagonal Fuzzy Set (PFS) ($\tilde{\mathcal{P}}$)

3.2.1 Set operation on PFS

Assume, $\tilde{\mathcal{A}} = \{\langle \xi; (\rho_1, \rho_2, \rho_3, \rho_4, \rho_5), \phi \rangle; \mu_{\tilde{\mathcal{A}}}(\xi)\}$ and $\tilde{\mathcal{B}} = \{\langle \xi; (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5), \psi \rangle; \mu_{\tilde{\mathcal{B}}}(\xi)\}$ are two PFSs define on \mathcal{X} . Then the various set operations on PFSs are defined as follows

a). Complement of PFS $\tilde{\mathcal{A}}$:

$$\tilde{\mathcal{A}}^c = \{\langle \xi; (\rho_1, \rho_2, \rho_3, \rho_4, \rho_5), 1 - \phi \rangle; \mu_{\tilde{\mathcal{A}}^c}(\xi)\} \quad (5)$$

b). Union of two PFSs $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$:

$$\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}} = \{\langle \xi; (\max\{\rho_1, \sigma_1\}, \max\{\rho_2, \sigma_2\}, \max\{\rho_3, \sigma_3\}, \max\{\rho_4, \sigma_4\}, \max\{\rho_5, \sigma_5\}), \max\{\phi, \psi\} \rangle; \mu_{\tilde{\mathcal{A}} \cup \tilde{\mathcal{B}}}(\xi)\} \quad (6)$$

c). Intersection of two PFSs $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$:

$$\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}} = \{ \langle \xi; (\min\{\rho_1, \sigma_1\}, \min\{\rho_2, \sigma_2\}, \min\{\rho_3, \sigma_3\}, \min\{\rho_4, \sigma_4\}, \min\{\rho_5, \sigma_5\}), \min\{\phi, \psi\} \rangle; \mu_{\tilde{\mathcal{A}} \cap \tilde{\mathcal{B}}}(\xi) \} \quad (7)$$

Example 3. Consider two PFSs $\tilde{\mathcal{G}} = \{ \langle \xi; (1, 2, 3, 4, 5), 0.75 \rangle; \mu_{\tilde{\mathcal{G}}}(\xi) \}$ and $\tilde{\mathcal{H}} = \{ \langle \xi; (2, 4, 5, 6, 8), 0.60 \rangle; \mu_{\tilde{\mathcal{H}}}(\xi) \}$ define in the universal set \mathcal{Y} . Then the basic set operations on fuzzy sets are

1. Complement of PFS $\tilde{\mathcal{G}}$:

$$\tilde{\mathcal{G}}^c = \{ \langle \xi; (1, 2, 3, 4, 5), 0.25 \rangle; \mu_{\tilde{\mathcal{G}}^c}(\xi) \}$$

2. Union of two PFSs $\tilde{\mathcal{G}}$ and $\tilde{\mathcal{H}}$:

$$\tilde{\mathcal{G}} \cup \tilde{\mathcal{H}} = \{ \langle \xi; (2, 4, 5, 6, 8), 0.75 \rangle; \mu_{\tilde{\mathcal{G}} \cup \tilde{\mathcal{H}}}(\xi) \}$$

3. Intersection of two PFSs $\tilde{\mathcal{G}}$ and $\tilde{\mathcal{H}}$:

$$\tilde{\mathcal{G}} \cap \tilde{\mathcal{H}} = \{ \langle \xi; (1, 2, 3, 4, 5), 0.60 \rangle; \mu_{\tilde{\mathcal{G}} \cap \tilde{\mathcal{H}}}(\xi) \}$$

3.3 Pentagonal Fuzzy Number (PFN)

Pentagonal fuzzy number (PFN) $\tilde{\mathcal{J}}$ is a pentagonal fuzzy set (PFS) defined in Definition 3 with satisfies the fuzzy numbers properties defined in Definition 2. Then the PFN is defined as

Definition 4. Pentagonal Fuzzy Number: [11]

Consider the set of real numbers (\mathbb{R}) be a universal set of discourse. A pentagonal fuzzy number (PFN) $\tilde{\mathcal{J}}$ is a PFS define as

$$\tilde{\mathcal{J}} = \{ \langle \zeta; (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5), \lambda \rangle; \mu_{\tilde{\mathcal{J}}}(\zeta) \} \quad (8)$$

where $\mu_{\tilde{\mathcal{J}}}$ be the membership function define as

$$\mu_{\tilde{\mathcal{J}}}(\zeta) = \begin{cases} 0 & \text{when } \zeta < \tau_1 \\ \lambda \frac{\zeta - \tau_1}{\tau_2 - \tau_1} & \text{when } \tau_1 \leq \zeta < \tau_2 \\ \lambda + (1 - \lambda) \frac{\zeta - \tau_2}{\tau_3 - \tau_2} & \text{when } \tau_2 \leq \zeta < \tau_3 \\ 1 & \text{when } \zeta = \tau_3 \\ \lambda + (1 - \lambda) \frac{\tau_4 - \zeta}{\tau_4 - \tau_3} & \text{when } \tau_3 < \zeta \leq \tau_4 \\ \lambda \frac{\tau_5 - \zeta}{\tau_5 - \tau_4} & \text{when } \tau_4 < \zeta \leq \tau_5 \\ 0 & \text{when } \tau_5 < \zeta \end{cases} \quad (9)$$

and λ are two extreme points of the pentagon with $0 \leq \lambda \leq 1$.

Based on the conditions on $\tau_i; i = 1, 2, \dots, 5$ the pentagonal fuzzy number ($\tilde{\mathcal{J}}$) convert into different types of fuzzy number, as

1. If $\tau_1 = \tau_2, \tau_2 = \tau_3, \tau_3 = \tau_4$ or $\tau_4 = \tau_5$, then the PFN $\tilde{\mathcal{J}}$ becomes trapezoidal fuzzy number (TrFN).

2. If $\tau_1 = \tau_2 = \tau_3, \tau_2 = \tau_3 = \tau_4$ or $\tau_3 = \tau_4 = \tau_5$, then the PFN $\tilde{\mathcal{F}}$ becomes triangular fuzzy number (TFN).

Furthermore, the extreme point λ of pentagonal fuzzy number ($\tilde{\mathcal{F}}$) lies between $[0, 1]$ and based on the conditions on λ the PFN can be restructured as follows

1. If $\lambda = 0$, then PFN $\tilde{\mathcal{F}}$ becomes TFN.
2. If $\lambda = 1$, then PFN $\tilde{\mathcal{F}}$ become TrFN.

Example 4. Let us, $\tilde{\mathcal{A}}$ be a PFN defined on a universal set of discourse \mathbb{R} . Then PFN $\tilde{\mathcal{A}}$ represent as

$$\tilde{\mathcal{A}} = \{ \langle \zeta, (3, 5, 7, 9, 11), 0.6 \rangle ; \mu_{\tilde{\mathcal{A}}}(\zeta) \}$$

where the membership function ($\mu_{\tilde{\mathcal{A}}}(\zeta)$) is a pentagonal shape bounded quintuple shown in above and by real line. The maximum membership value spotted at the point is 7 and the other two corner points are 5 and 9.

3.3.1 Arithmetic Operations on PFN

Assume, $\tilde{\mathcal{E}} = \{ \langle \xi; (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5), \phi \rangle ; \mu_{\tilde{\mathcal{E}}}(\xi) \}$ and $\tilde{\mathcal{D}} = \{ \langle \xi; (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5), \psi \rangle ; \mu_{\tilde{\mathcal{D}}}(\xi) \}$ are two PFNs define on \mathbb{R} . Then the various arithmetic operations on PFNs [10] are defined as follows

- a). Addition of two PFNs:

$$\tilde{\mathcal{E}} \oplus \tilde{\mathcal{D}} = \{ \langle \xi; (\tau_1 + \sigma_1, \tau_2 + \sigma_2, \tau_3 + \sigma_3, \tau_4 + \sigma_4, \tau_5 + \sigma_5), \max\{\phi, \psi\} \rangle ; \mu_{\tilde{\mathcal{E}} \oplus \tilde{\mathcal{D}}}(\xi) \} \quad (10)$$

- b). Subtraction from PFN to PFN:

$$\tilde{\mathcal{E}} \ominus \tilde{\mathcal{D}} = \{ \langle \xi; (\tau_1 - \sigma_5, \tau_2 - \sigma_4, \tau_3 - \sigma_3, \tau_4 - \sigma_2, \tau_5 - \sigma_1), \min\{\phi, \psi\} \rangle ; \mu_{\tilde{\mathcal{E}} \ominus \tilde{\mathcal{D}}}(\xi) \} \quad (11)$$

- c). Scalar multiplication of PFN:

$$\delta \tilde{\mathcal{E}} = \{ \langle \xi; (\delta\tau_1, \delta\tau_2, \delta\tau_3, \delta\tau_4, \delta\tau_5), \delta\phi \rangle ; \mu_{\delta\tilde{\mathcal{E}}}(\xi) \} \quad (12)$$

where δ is a positive scalar number.

- d). Multiplication of two PFNs:

$$\tilde{\mathcal{E}} \otimes \tilde{\mathcal{D}} = \{ \langle \xi; (\tau_1 \times \sigma_1, \tau_2 \times \sigma_2, \tau_3 \times \sigma_3, \tau_4 \times \sigma_4, \tau_5 \times \sigma_5), \max\{\phi, \psi\} \rangle ; \mu_{\tilde{\mathcal{E}} \otimes \tilde{\mathcal{D}}}(\xi) \} \quad (13)$$

- e). Scalar power of PFN:

$$\tilde{\mathcal{E}}^\kappa = \{ \langle \xi; (\tau_1^\kappa, \tau_2^\kappa, \tau_3^\kappa, \tau_4^\kappa, \tau_5^\kappa), \phi^\kappa \rangle ; \mu_{\tilde{\mathcal{E}}^\kappa}(\xi) \} \quad (14)$$

where κ is a positive scalar number.

More details explanation of arithmetic operation on fuzzy numbers is discoursed by Mukherjee, A. K. et al. [2]. This study explained the limitations and conditions for arithmetic operations on fuzzy sets and fuzzy numbers are mentioned with graphical representation.

Example 5. Let us assume, two PFNs $\tilde{\mathcal{A}} = \{ \langle \xi; (6, 7, 9, 11, 12), 0.60 \rangle ; \mu_{\tilde{\mathcal{A}}}(\xi) \}$ and $\tilde{\mathcal{B}} = \{ \langle \xi; (3, 5, 6, 7, 9), 0.80 \rangle ; \mu_{\tilde{\mathcal{B}}}(\xi) \}$ define in the universal set \mathbb{R} . Then the arithmetic operations on fuzzy numbers are

1. Addition of two PFNs $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$:

$$\tilde{\mathcal{A}} \oplus \tilde{\mathcal{B}} = \{ \langle \xi; (9, 12, 15, 18, 21), 0.80 \rangle ; \mu_{\tilde{\mathcal{A}} \oplus \tilde{\mathcal{B}}}(\xi) \}$$

2. Subtraction from PFN $\tilde{\mathcal{A}}$ to PFN $\tilde{\mathcal{B}}$:

$$\tilde{\mathcal{A}} \ominus \tilde{\mathcal{B}} = \{ \langle \xi; (-3, 0, 3, 6, 9), 0.60 \rangle ; \mu_{\tilde{\mathcal{A}} \ominus \tilde{\mathcal{B}}}(\xi) \}$$

3. Scalar multiplication of PFN $\tilde{\mathcal{A}}$ by 0.5:

$$0.5\tilde{\mathcal{A}} = \{ \langle \xi; (3, 3.5, 4.5, 5.5, 6), 0.30 \rangle ; \mu_{0.5\tilde{\mathcal{A}}}(\xi) \}$$

here $\delta = 0.5$ is a positive scalar number.

4. Multiplication of two PFNs $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$:

$$\tilde{\mathcal{A}} \otimes \tilde{\mathcal{B}} = \{ \langle \xi; (18, 35, 54, 77, 108), 0.80 \rangle ; \mu_{\tilde{\mathcal{A}} \otimes \tilde{\mathcal{B}}}(\xi) \}$$

5. Scalar power of PFN:

$$\tilde{\mathcal{C}}^{0.5} = \{ \langle \xi; (2.4495, 2.6458, 3.0000, 3.3166, 3.4641), 0.77 \rangle ; \mu_{\tilde{\mathcal{C}}^{0.5}}(\xi) \}$$

here $\kappa = 0.5$ is a positive scalar number.

3.4 De-fuzzification of PFN

De-fuzzification [54] is a computational procedure to determine an equivalent crisp number for every fuzzy number. The fuzzification and de-fuzzification are two contradictory procedures for crisp to fuzzy numbers and vice versa, respectively. In the fuzzy number system, there is no order relation but in the crisp number system have, so capsizes the fuzzy number to get an order relation. Several methodologies already invented for de-fuzzified the fuzzy numbers, such as centre of area (COA) method, bisector of area (BOA) method, largest of maxima (LOM) method, smallest of maxima (SOM) method, mean of maximum (MOM) method, regular weighted point (RWP) method, graded mean integration value (GMIV) method, centre of approximated interval (COAI) method, centroid based method (CBM) method and so on. Here, we proposed a new method on PFN to de-fuzzify PFN.

Assume, $\tilde{\mathcal{C}} = \{ \langle \zeta; (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5), \lambda \rangle ; \mu_{\tilde{\mathcal{C}}}(\zeta) \}$ be a PFN define on the real line (\mathbb{R}). Then the proposed de-fuzzification method [11] on PFN ($\tilde{\mathcal{C}}$) evaluated as follows:

$$\mathcal{D}(\tilde{\mathcal{C}}) = \frac{(\tau_1 + 2 \times \tau_2 + 3 \times \tau_3 + 2 \times \tau_4 + \tau_5)(1 + \lambda)}{9} \tag{15}$$

Example 6. Consider two PFNs $\tilde{\mathcal{S}} = \{ \langle \zeta; (1, 2, 3, 4, 5), 0.6 \rangle ; \mu_{\tilde{\mathcal{S}}}(\zeta) \}$ and $\tilde{\mathcal{T}} = \{ \langle \zeta; (2, 4, 6, 8, 10), 0.75 \rangle ; \mu_{\tilde{\mathcal{T}}}(\zeta) \}$ then the de-fuzzified values of $\tilde{\mathcal{S}}$ and $\tilde{\mathcal{T}}$ is denoted by $\mathcal{D}(\tilde{\mathcal{S}})$ and $\mathcal{D}(\tilde{\mathcal{T}})$, respectively and determine as

$$\begin{aligned} \mathcal{D}(\tilde{\mathcal{S}}) &= \frac{(1 + 2 \times 2 + 3 \times 3 + 2 \times 4 + 5)(1 + 0.60)}{9} = \frac{43.2}{9} = 4.8 \\ \mathcal{D}(\tilde{\mathcal{T}}) &= \frac{(2 + 2 \times 4 + 3 \times 6 + 2 \times 8 + 10)(1 + 0.75)}{9} = \frac{94.5}{9} = 10.5 \end{aligned}$$

4. Fuzzy decision making trial and evaluation laboratory (FDEMATEL) method

This section briefly explains the Decision making trial and evaluation laboratory (DEMATEL) method in a pentagonal fuzzy number (PFN) environment. This method was first invented in 2007 by Tzeng, G. et al. [18]. The DEMATEL method has been extensively utilised in numerous fields, such as risk management [19], airline safety oversight [20], web-advertising impact [17], auditing in a setting of enterprise resource planning [21], the effect of e-learning programs [18], investigating hospital service quality [22], deciding on the quality model’s cost [14], selecting the provider for a mobile banking service [55], risk factors evaluation in automobile industry [56], and so on. Here, the DEMATEL method is applied in a pentagonal fuzzy number (PFN) uncertain environment and called the FDEMATEL method. The structural flowchart of the FDEMATEL methods is graphically presented in Figure 4. The fuzzy DEMATEL (FDEMATEL) method is processed as follows:

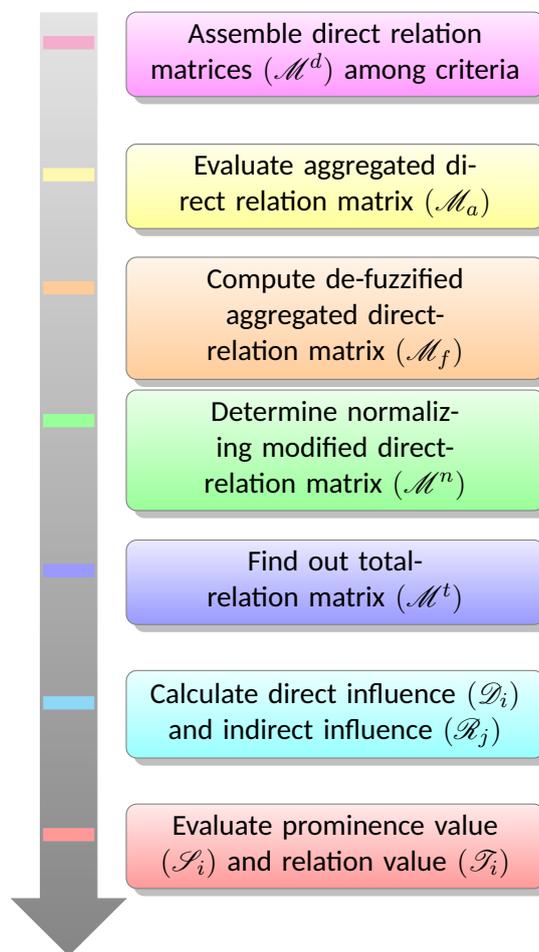


Fig. 4. Hierarchical structure of the DEMATEL method

I. Construct the direct relation matrix (\mathcal{M}^d):

The estimation of the interrelation between i th criteria with j th criteria that measured by comparison scale by Table 1 where $i, j = 1, 2, \dots, p$. There are s number of decision makers (DMs) who give opinions based on their experience and knowledge. There are s direct relation matrices (\mathcal{M}^d) formulated with $p \times p$ order where $d = 1, 2, \dots, s$. The d th direct relation matrix

(\mathcal{M}^d) structured as

$$\mathcal{M}^d = \begin{bmatrix} (\tilde{\mathcal{A}}_{11})^d & (\tilde{\mathcal{A}}_{12})^d & \dots & (\tilde{\mathcal{A}}_{1j})^d & \dots & (\tilde{\mathcal{A}}_{1p})^d \\ (\tilde{\mathcal{A}}_{21})^d & (\tilde{\mathcal{A}}_{22})^d & \dots & (\tilde{\mathcal{A}}_{2j})^d & \dots & (\tilde{\mathcal{A}}_{2p})^d \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (\tilde{\mathcal{A}}_{i1})^d & (\tilde{\mathcal{A}}_{i2})^d & \dots & (\tilde{\mathcal{A}}_{ij})^d & \dots & (\tilde{\mathcal{A}}_{ip})^d \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (\tilde{\mathcal{A}}_{p1})^d & (\tilde{\mathcal{A}}_{p2})^d & \dots & (\tilde{\mathcal{A}}_{pj})^d & \dots & (\tilde{\mathcal{A}}_{pp})^d \end{bmatrix}_{p \times p} \quad (16)$$

where $(\tilde{\mathcal{A}}_{ij})^d$ entry represents the ratings of j th criteria respect to i th criteria given by d th DM. Additionally, $(\tilde{\mathcal{A}}_{ij})^d$ is a pentagonal fuzzy number represented as

$$\begin{aligned} (\tilde{\mathcal{A}}_{ij})^d &= \left(\left(\langle \zeta; (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5), \lambda \rangle; \mu_{\tilde{\mathcal{A}}}(\zeta) \right)_{ij} \right)^d \\ &= \left\{ \langle (\zeta)_{ij}^d; ((\tau_1)_{ij}^d, (\tau_2)_{ij}^d, (\tau_3)_{ij}^d, (\tau_4)_{ij}^d, (\tau_5)_{ij}^d), (\lambda)_{ij}^d \rangle; (\mu_{\tilde{\mathcal{A}}}(\zeta))_{ij}^d \right\} \\ &= \left(\left\langle (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5), \lambda \right\rangle_{ij}; \mu_{\tilde{\mathcal{A}}}(\zeta)_{ij} \right)^d \end{aligned} \quad (17)$$

II. Aggregated the direct relation matrices (\mathcal{M}_a) :

All s number of direct matrices (\mathcal{M}^d) where $d = 1, 2, \dots, s$, merge into one direct relation matrix (\mathcal{M}_a) for further evaluation. Merge the every ij th entry $(\tilde{\mathcal{A}}_{ij})$ by Equation (18) and construct single direct relation matrix (\mathcal{M}_a) as follows:

$$\begin{cases} (\tau_1)_{ij} = \min_{d=1,2,\dots,s} \{ (\tau_1)_{ij}^d \} \\ (\tau_2)_{ij} = \min_{d=1,2,\dots,s} \{ (\tau_2)_{ij}^d \} \\ (\tau_3)_{ij} = \sqrt[s]{\prod_{d=1}^s (\tau_3)_{ij}^d} \\ (\tau_4)_{ij} = \max_{d=1,2,\dots,s} \{ (\tau_4)_{ij}^d \} \\ (\tau_5)_{ij} = \max_{d=1,2,\dots,s} \{ (\tau_5)_{ij}^d \} \\ (\lambda)_{ij} = \max_{d=1,2,\dots,s} \{ (\lambda)_{ij}^d \} \end{cases} \quad (18)$$

and direct relation matrix (\mathcal{M}_a) looks like,

$$\mathcal{M}_a = [\tilde{\mathcal{A}}_{ij}]_{p \times p} = \begin{bmatrix} \tilde{\mathcal{A}}_{11} & \tilde{\mathcal{A}}_{12} & \dots & \tilde{\mathcal{A}}_{1j} & \dots & \tilde{\mathcal{A}}_{1p} \\ \tilde{\mathcal{A}}_{21} & \tilde{\mathcal{A}}_{22} & \dots & \tilde{\mathcal{A}}_{2j} & \dots & \tilde{\mathcal{A}}_{2p} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\mathcal{A}}_{i1} & \tilde{\mathcal{A}}_{i2} & \dots & \tilde{\mathcal{A}}_{ij} & \dots & \tilde{\mathcal{A}}_{ip} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\mathcal{A}}_{p1} & \tilde{\mathcal{A}}_{p2} & \dots & \tilde{\mathcal{A}}_{pj} & \dots & \tilde{\mathcal{A}}_{pp} \end{bmatrix}_{p \times p} \quad (19)$$

III. De-fuzzified the aggregated direct-relation matrix (\mathcal{M}_f) :

Determine the de-fuzzified aggregated direct-relation matrix (\mathcal{M}_f) by de-fuzzified every entry of the aggregated direct-relation matrix (\mathcal{M}_a) using the de-fuzzified formula in Equation (15). The de-fuzzified aggregated direct-relation matrix (\mathcal{M}_f) formulate as

$$\mathcal{M}_f = [\mathcal{A}_{ij}]_{p \times p} \quad (20)$$

where \mathcal{A}_{ij} is the de-fuzzified value of PFN $\tilde{\mathcal{A}}_{ij}$.

IV. Normalizing the modified direct-relation matrix (\mathcal{M}^n):

Further, normalized the modified direct-relation matrix \mathcal{M}_f by using Equation (21) and establish the normalizing direct-relation matrix (\mathcal{M}^n) as follows:

$$\mathcal{M}^n = [\mathcal{A}_{ij}^n]_{p \times p} = \left[\frac{\mathcal{A}_{ij}}{\max_{1 \leq i \leq p} \left\{ \sum_{j=1}^p \mathcal{A}_{ij} \right\}} \right]_{p \times p} \quad (21)$$

where \mathcal{A}_{ij}^n is the normalized value of \mathcal{A}_{ij} and $i, j = 1, 2, \dots, p$.

V. Evaluate the total-relation matrix (\mathcal{M}^t):

Calculate the total-relation matrix (\mathcal{M}^t) from the normalizing direct-relation matrix (\mathcal{M}^n) as follows:

$$\begin{aligned} \mathcal{M}^t &= \lim_{t \rightarrow \infty} \langle \mathcal{M}^n + (\mathcal{M}^n)^2 + (\mathcal{M}^n)^3 + \dots + (\mathcal{M}^n)^t \rangle \\ \text{or, } \mathcal{M}^t &= \mathcal{M}^n (\mathcal{I} - \mathcal{M}^n)^{-1} = [\mathcal{B}_{ij}]_{p \times p} \end{aligned} \quad (22)$$

where \mathcal{I} be the identity matrix of p th order and \mathcal{B}_{ij} is the ij th entry of the total-relation matrix (\mathcal{M}^t).

VI. Determine direct influence (\mathcal{D}_i) and indirect influence (\mathcal{R}_j) for the criteria:

The sum of i th row entries of the total-relation matrix (\mathcal{M}^t) is denoted direct influence value (\mathcal{D}_i) and describe as

$$\mathcal{D}_i = \sum_{j=1}^p \mathcal{B}_{ij} \quad (23)$$

where $i = 1, 2, \dots, p$ and the sum of j th column entries of the total-relation matrix (\mathcal{M}^t) is denoted indirect influence value (\mathcal{R}_j) and describe as

$$\mathcal{R}_j = \sum_{i=1}^p \mathcal{B}_{ij} \quad (24)$$

where $j = 1, 2, \dots, p$.

VII. Evaluate sum values:

Causal diagrams for every criteria are evaluated from the data set. The prominence value (\mathcal{S}_i) of every criteria are lies in the horizontal axis and determined as follows:

$$\mathcal{S}_i = \mathcal{D}_i + \mathcal{R}_i = \sum_{j=1}^p \mathcal{B}_{ij} + \sum_{j=1}^p \mathcal{B}_{ji} \quad (25)$$

and the relation value (\mathcal{F}_i) of every criteria are lies in the vertical axis and determined as follows:

$$\mathcal{F}_i = \mathcal{D}_i - \mathcal{R}_i = \sum_{j=1}^p \mathcal{B}_{ij} - \sum_{j=1}^p \mathcal{B}_{ji} \quad (26)$$

where \mathcal{D}_i and \mathcal{R}_i are evaluated in previous steps where $i = 1, 2, \dots, p$.

VIII. Calculate threshold value:

Finally, \mathcal{S}_i values revealed how much important the criteria is, that is it exhibits the degree of the relation of every criteria with the remaining criteria. Therefore, the criteria have higher value of \mathcal{S}_i indicates that the criteria have more relation with other criteria and simultaneously, those criteria have lower value of \mathcal{S}_i displayed that the criteria have less relation with other criteria.

On the other side, \mathcal{T}_i value unfolds that the kind of relation among the criteria. When the \mathcal{T}_i value is positive (*+ve*) then the criteria belongs to the cause group or dispatcher, that is the i th criteria influence other criteria. Alternatively, when the \mathcal{T}_i value is negative (*-ve*) then the criteria belong to the effect group or receiver, that is i th criteria influenced by other criteria.

Lastly, based on the \mathcal{S}_i and \mathcal{T}_i values of the criteria, drawn the causal diagram in XY plane, where $i = 1, 2, \dots, p$. The causal diagram shows the direct and indirect effect of each criteria over the other criteria. Furthermore, decision makers can set up the threshold value to identify the negligible effect of criteria over all other criteria.

5. Model formulation and Data collection

This section describes this study's model formulation and data collection process in detail. First, the model of the proposed research will be formulated, followed by the data assembly procedure from three decision experts.

5.1 Model formulation

There are six most important criteria for finding valuable criteria in the sports section based on women's empowerment, which are considered according to decision makers' opinions. The detailed discussion on criteria is mentioned in Section 2. The uncertainty capturing mathematical tool is discussed in Section 3 and the fuzzy DEMATEL based MCDM method is mentioned in Section 4, respectively. The direct relation matrices (\mathcal{M}^d) with 6×6 order in linguistic terms given by three DMs. The structural flowchart of the proposed model is depicted in Figure 5. All those DMs are experienced, unbiased and professional in their field and they are:

DM1: A women's international sports champion.

DM2: An officer in sports authority board.

DM3: A social worker works on women's empowerment.



Fig. 5. Hierarchical structure of the proposed model

5.2 Data collection

This section describes the data collection of this study. Firstly, the data are collected by DMs in linguistic terms in the direct relation matrices (\mathcal{M}^d) using Table 1 and contained in Table 2. Further, the direct relation matrices (\mathcal{M}^d) decoded into PFNs using Table 1 and initiated for the fuzzy DEMATEL method using Section 4.

Table 1
 Linguistic term with GIPFN and de-fuzzified value for decision matrix

Linguistic Terms	Pentagonal Fuzzy Number (PFN)	De-fuzzified Value
Extremely Relevant (ER)	$\langle (10, 11, 13, 15, 16), 2/3 \rangle$	13
High Relevant (HR)	$\langle (8, 9, 11, 13, 14), 2/3 \rangle$	11
Very Relevant (VR)	$\langle (6, 7, 9, 11, 12), 2/3 \rangle$	9
Moderate Relevant (MR)	$\langle (4, 5, 7, 9, 10), 2/3 \rangle$	7
Less Relevant (LR)	$\langle (2, 3, 5, 7, 8), 2/3 \rangle$	5
Below Relevant (BR)	$\langle (0, 1, 3, 5, 6), 2/3 \rangle$	3
Weekly Relevant (WR)	$\langle (-2, -1, 1, 3, 4), 2/3 \rangle$	1
No Relevant (NR)	$\langle (-3, -2, 0, 2, 3), 2/3 \rangle$	0

Table 2
 Direct relation matrix (\mathcal{M}^d) in linguistic terms given by three DMs

	Criteria vs Criteria	Supportive Infrastructure (S_1)	Role Models and Representation (S_2)	Policy and Governance (S_3)	Community and Cultural Acceptance (S_4)	Support Systems (Family and financial) (S_5)	Support Systems (Family and financial) (S_6)
DM 1	Supportive Infrastructure (S_1)	NR	LR	HR	VR	HR	ER
	Role Models and Representation (S_2)	BR	NR	WR	MR	VR	LR
	Policy and Governance (S_3)	WR	HR	NR	HR	MR	BR
	Community and Cultural Acceptance (S_4)	VR	LR	HR	NR	ER	VR
	Support Systems (Family and financial) (S_5)	MR	HR	VR	WR	NR	LR
	Community and Cultural Acceptance (S_6)	VR	MR	HR	BR	VR	NR
DM 2	Supportive Infrastructure (S_1)	NR	MR	ER	HR	ER	HR
	Role Models and Representation (S_2)	LR	NR	BR	VR	HR	MR
	Policy and Governance (S_3)	BR	ER	NR	ER	VR	LR
	Community and Cultural Acceptance (S_4)	HR	MR	ER	NR	HR	HR
	Support Systems (Family and financial) (S_5)	VR	ER	HR	BR	NR	MR
	Community and Cultural Acceptance (S_6)	HR	VR	ER	LR	HR	NR
DM 3	Supportive Infrastructure (S_1)	NR	BR	VR	MR	VR	HR
	Role Models and Representation (S_2)	WR	NR	BR	LR	MR	BR
	Policy and Governance (S_3)	BR	VR	NR	VR	LR	WR
	Community and Cultural Acceptance (S_4)	MR	BR	VR	NR	HR	MR
	Support Systems (Family and financial) (S_5)	LR	VR	MR	BR	NR	BR
	Community and Cultural Acceptance (S_6)	MR	LR	VR	WR	MR	NR

Remark 2. Table 2 represented the direct relation matrix (\mathcal{M}^d) in linguistic terms by three DMs using Table 1. All the ratings are taken in an unbiased way and forwarded for numerical evaluation.

6. Numerical illustration and discussion

The numerical evaluation of the most effective criteria for women’s empowerment through sports is determined and the results are analysed here, in detail. We consider the fuzzy DEMATEL methodology (see Section 4) as an optimization technique in Pentagonal Fuzzy Number (PFN) (see Section 3) as an uncertain environment. Further, consider the Direct relation matrix (\mathcal{M}^d) (in Table 2) as the data source for numerical evaluation.

The fuzzy DEMATEL method evaluated the required results as follows: Firstly, construct the direct relation matrix (\mathcal{M}^d) by three DMs using Equation (16), shown in Table 2. Then aggregated direct relation matrices (\mathcal{M}_a) evaluated by Equation (19). Further, the de-fuzzified aggregated direct-relation matrix (\mathcal{M}_f) determined by Equation (20) and shown in Table 3. Thereafter, Equation (21) calculated the normalizing modified direct-relation matrix (\mathcal{M}^n) in Table 4. Then evaluate the total-relation matrix (\mathcal{M}^t) in Table 5 using Equation (22). Finally, the direct influence (\mathcal{D}_i), indirect influence (\mathcal{R}_j),

prominence value (\mathcal{S}_i) and relation value (\mathcal{R}_i) values for every criteria ($i = 1, 2, \dots, p$) are calculated by Equations (23), (24), (25) and (26), respectively and shown in Table 6.

Table 3
 De-fuzzified the aggregated direct-relation matrix (\mathcal{M}_f)

Criteria vs criteria	S ₁	S ₂	S ₃	S ₄	S ₅	S ₆
Supportive Infrastructure (S_1)	0.0000	8.1765	18.2652	14.9163	18.2652	19.7944
Role Models and Representation (S_2)	4.7035	0.0000	3.3778	11.5578	14.9163	8.1765
Policy and Governance (S_3)	3.3778	18.2652	0.0000	18.2652	11.5578	4.7035
Community and Cultural Acceptance (S_4)	14.9163	8.1765	18.2652	0.0000	19.7944	14.9163
Support Systems (Family and financial) (S_5)	11.5578	18.2652	14.9163	3.3778	0.0000	8.1765
Community and Cultural Acceptance (S_6)	14.9163	11.5578	18.2652	4.7035	14.9163	0.0000

Table 4
 Normalizing the modified direct-relation matrix (\mathcal{M}^n)

Criteria vs criteria	S ₁	S ₂	S ₃	S ₄	S ₅	S ₆
Supportive Infrastructure (S_1)	0.0000	0.1030	0.2300	0.1878	0.2300	0.2492
Role Models and Representation (S_2)	0.0592	0.0000	0.0425	0.1455	0.1878	0.1030
Policy and Governance (S_3)	0.0425	0.2300	0.0000	0.2300	0.1455	0.0592
Community and Cultural Acceptance (S_4)	0.1878	0.1030	0.2300	0.0000	0.2492	0.1878
Support Systems (Family and financial) (S_5)	0.1455	0.2300	0.1878	0.0425	0.0000	0.1030
Community and Cultural Acceptance (S_6)	0.1878	0.1455	0.2300	0.0592	0.1878	0.0000

Table 5
 Total-relation matrix (\mathcal{M}^t)

Criteria vs criteria	S ₁	S ₂	S ₃	S ₄	S ₅	S ₆
Supportive Infrastructure (S_1)	0.4752	0.7262	0.8569	0.6652	0.9070	0.7099
Role Models and Representation (S_2)	0.3350	0.3609	0.4226	0.4108	0.5740	0.3807
Policy and Governance (S_3)	0.3772	0.6386	0.4574	0.5533	0.6439	0.4151
Community and Cultural Acceptance (S_4)	0.6152	0.7040	0.8305	0.4881	0.8946	0.6462
Support Systems (Family and financial) (S_5)	0.4486	0.6367	0.6086	0.4071	0.5007	0.4397
Community and Cultural Acceptance (S_6)	0.5394	0.6522	0.7269	0.4819	0.7454	0.4098

Table 6
 Threshold values of the different criteria

Criteria	$\mathcal{D}_i = \sum_{j=1}^p \mathcal{B}_{ij}$	$\mathcal{R}_j = \sum_{i=1}^p \mathcal{B}_{ij}$	$\mathcal{S}_i = \mathcal{D}_i + \mathcal{R}_i$	$\mathcal{T}_i = \mathcal{D}_i - \mathcal{R}_i$
Supportive Infrastructure (S_1)	4.3404	2.7906	7.1310	1.5498
Role Models and Representation (S_2)	2.4840	3.7186	6.2026	-1.2346
Policy and Governance (S_3)	3.0855	3.9029	6.9884	-0.8174
Community and Cultural Acceptance (S_4)	4.1786	3.0064	7.1850	1.1722
Support Systems (Family and financial) (S_5)	3.0414	4.2656	7.3070	-1.2242
Community and Cultural Acceptance (S_6)	3.5556	3.0014	6.5570	0.5542

Table 6 represented the numerical results of the fuzzy DEMATEL method between criteria of women empowerment through sports. From the prominence value (\mathcal{S}_i) values, we conclude that the degree of the relationship of one particular criterion among the other criteria. For example, the criteria Support Systems (Family and financial) (S_5) is the maximum relation with other criteria. Further, Community and Cultural Acceptance (S_4) and Supportive Infrastructure (S_1) criteria are the second and third height relation with the remaining criteria, respectively. Additionally, Policy and Governance (S_3) and Community and Cultural Acceptance (S_6) criteria are the fourth and fifth highest relation and the Role Models and Representation (S_2) criteria is the lowest relation among the criteria, respectively.

Besides that, the relation value (\mathcal{T}_i) values of the criteria are also known from Table 6 and it reveals the kind of relation among the criteria. The relation value (\mathcal{T}_i) values positive (+ve) indicates that the criteria belong to the cause group or dispatcher and negative (-ve) indicates that the criteria belong to the effect group or receiver. For example, Supportive Infrastructure (S_1), Community and Cultural Acceptance (S_4) and Community and Cultural Acceptance (S_6) criteria are the cause group or dispatcher and Policy and Governance (S_3), Support Systems (Family and financial) (S_5) and Role Models and Representation (S_2) criteria are the effect group or receiver, respectively. The causal diagram among the criteria of women’s empowerment through sports is depicted in Figure 6.

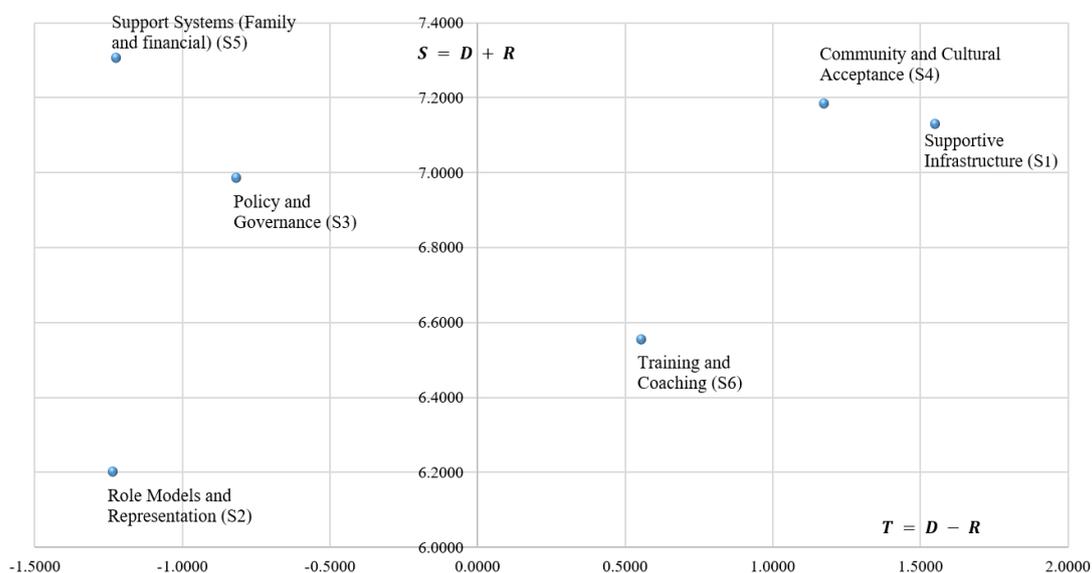


Fig. 6. Causal diagram for every criterion based on \mathcal{S}_i and \mathcal{T}_i values

7. Conclusion and future research scope

In this article, we find the direct and indirect effects of every criteria over the other criteria of some related criteria for women empowerment in the sports sector. The criteria are taken as Supportive Infrastructure (S_1), Role Models and Representation (S_2), Policy and Governance (S_3), Community and Cultural Acceptance (S_4), Support Systems (Family and financial) (S_5) and Training and Coaching (S_6). we use the concept of multi criteria decision making to find the direct and indirect effects of the criteria. The data sets are collected in terms of linguistic variables from three decision experts. The linguistic terms direct relation matrix (\mathcal{M}^d) converted to a fuzzy number matrix for further steps. The method namely, fuzzy DEMATEL is taken to find the numerical results in Table 6. In conclusion, the criteria Support Systems (Family and financial) (S_5) is the highest relation and the Role Models and Representation (S_2) is the least relation with the remaining criteria. Furthermore, the criteria (S_1),

(S_4) and (S_6) are in the cause group or dispatcher and the remaining criteria (S_3) , (S_5) and (S_2) are in effect group or receiver, respectively.

Through the organized approach, such studies offer important understandings of pretty opportunities, nurturing supportive environments and empowering women to shine in the sports sector. This research will eventually contribute to generating a more inclusive and equitable sports landscape for forthcoming generations. The consequences offer strategic visions for sports organizations, policymakers and stakeholders to implement targeted creativity that improves leadership roles, gender equity and complete empowerment for women in sports.

In future research, the work may be extended in two different ways, either from a theoretical perspective or in terms of model formulations. Anyone may take another type of MCDM method such as Analytical Hierarchy Process (AHP), Analytic Network Process (ANP), Entropy, CRiteria Importance Through Inter-criteria Correlation (CRITIC) and Stepwise Weight Assessment Ratio Analysis (SWARA) methods to find the relationship between the criteria and take different types of uncertainty in data sets like intuitionistic fuzzy sets, Pythagorean fuzzy sets, neutrosophic fuzzy sets, probabilistic linguistic term sets, etc. for better results. In the model portion, anyone may use the different perspectives of women empowerment in different sectors like education, social work, journalism, politics, etc.

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Conflicts of Interest

The authors declare that they have no known conflicts of interest or personal relationships that could have appeared to influence the work reported in this paper. There are no conflicts of interest between authors.

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