SCIENTIFIC OASIS

Spectrum of Decision Making and Applications

Journal homepage: www.dmap-journal.org
ISSN: 3042-1462

Sustainable Urban Innovation and Resilience: Artificial Intelligence and q-Rung Orthopair Fuzzy ExpoLogarithmic Framework

Subramanian Petchimuthu[1], Balakrishnan Palpandi[2,*], Pirabaharan P[3], Fathima Banu M[1]

[1] Department of Science and Humanities (Mathematics), University College of Engineering Nagercoil, Anna University, Tamilnadu, India
[2] Department of Mathematics, University College of Engineering, Bharathidasan Institute of Technology Campus, Anna University, Tamilnadu, India
[3] Department of Mathematics, Anna University, Regional Campus-Madurai, Keelakuilkudi, Tamilnadu, India

ARTICLE INFO

Article history:
Received 30 November 2024
Received in revised form11 January 2025
Accepted 26 January 2025
Available online 29 January 2025

Keywords:

Q-rung orthopair fuzzy sets; ExpoLogarithmic operations; MCDM

ABSTRACT

In the face of rapid urbanization and the challenges posed by climate change, achieving sustainability and resilience in urban environments has become imperative. This paper explores a novel approach integrating Artificial Intelligence, Multi-Criteria Decision Making (MCDM), and q-rung orthopair fuzzy ExpoLogarithmic aggregation operators to address the complex dynamics of sustainable urban development. The paper introduces novel ExpoLogarithmic operations rooted in ExpoLogarithmic t-norms within the framework of q-rung orthopair fuzzy sets. By harnessing these operations, the paper proposes two robust aggregation operators under q-rung orthopair fuzzy sets: the q-rung orthopair fuzzy ExpoLogarithmic weighted average and the q-rung orthopair fuzzy ExpoLogarithmic weighted geometric. These aggregation operators demonstrate fundamental properties, including idempotency, monotonicity, and boundedness. To assess the effectiveness of these methods, a MCDM methodology utilizing the recommended aggregation operators is employed. This study integrates a practical demonstration in real-time, with a specific emphasis on enacting appropriate policies for sustainable urban innovation and resilience. Comprehensive analyses, comprising sensitivity, comparative, and performance evaluations of the approaches, are carried out. Additionally, a reflective discourse on the advantages and disadvantages of the proposed aggregation operators is provided alongside the analysis, underscoring the importance of this approach in bolstering city resilience and mitigating environmental impact.

*Corresponding author.
E-mail address: palpandiphd15@gmail.com

https://doi.org/10.31181/sdmap21202526

1. Introduction

Sustainable urban innovation and resilience depend on leveraging Artificial Intelligence to tackle modern urban challenges. This approach optimizes resource use, enhances sustainability, and strengthens urban resilience, enabling smarter city planning and adaptive solutions. Embracing AI is essential for fostering innovation and long-term sustainability in cities facing evolving environmental and societal pressures. For the full form of abbreviations used in the paper, refer to Table 8 in the Appendix.

1.1 *Motivation for the proposed research*

Urban planning is undergoing a transformation with the rise of artificial intelligence (AI), offering the potential to redefine urban landscapes. However, challenges remain in understanding the full impact of AI on urban and regional planning, including the need for strategic responses [1]. As cities aim for sustainability, AI offers promising solutions for optimizing resource allocation, enhancing infrastructure efficiency, and fostering innovation. However, Allam et al. [2] caution against the uncritical adoption of AI, emphasizing the need for careful integration to ensure truly sustainable and resilient cities.

The Multi-Criteria Decision-Making (MCDM) approach is a powerful tool for evaluating multiple criteria, accommodating both quantitative and qualitative data, and ensuring transparency [3–5]. MCDM helps identify optimal solutions in various domains, including waste management, urban innovation, and site selection, involving three key steps: information gathering, aggregation of data, and ranking alternatives [6]. The ExpoLogarithmic t-norm is a flexible aggregation method in fuzzy logic that adapts to specific system requirements using the parameter μ. Key features of this t-norm include sensitivity to input values, logarithmic scaling, and non-linear aggregation, making it suitable for dynamic and adaptive systems. It maintains compatibility with traditional t-norms, converging to the min operator as μ increases. The motivation for using ExpoLogarithmic t-norms and t-conorms lies in their flexibility, interpretability, ability to handle nonlinearity, and applicability in various fuzzy logic applications [7]. These norms allow users to adjust the fuzziness or aggregation level to meet specific needs, offering more intuitive results and enhancing decision-making.

The q-rung orthopair fuzzy set (q-ROFS) extends orthopair fuzzy sets, providing a higher degree of granularity in representing uncertainties and preferences. q-ROFS is particularly useful in urban planning, environmental management, and decision-making under uncertainty. Inspired by work in sustainable energy planning [8], our research applies q-ROFS and ExpoLogarithmic operators to enhance sustainable urban innovation and resilience, addressing uncertainties in urban decision-making.

1.2 *Literature review*

Numerous conventional approaches in formal computing yield precise and binary outcomes, with results expressed as either a definite yes or no. This binary nature often struggles to capture the nuances inherent in information. To address this limitation, Zadeh introduced the Fuzzy Set (FS) in 1965 [9], wherein membership grades are assigned to potential individuals within the unit interval [0, 1]. However, the fuzzy set is solely delineated by its membership value (MV), which proves inadequate for numerous real-life decisions. Decision-makers must also furnish the nonmembership value (NMV) to evaluate attribute values effectively. Expanding on this idea, Atanassov [10] introduced the intuitionistic fuzzy set (IFS), which incorporates non-membership grades. Recent studies have extensively tackled various Multiple criteria Decision Making (MCDM) issues using Intuitionistic Fuzzy Sets (IFSs) [11, 12]. Seikh and Mandal [13] pioneered Dombi aggregation operators (AOs) to merge job information and determine the most favorable job employing intuitionistic fuzzy (IF) data. Senapati et al.[14] introduced IF Aczel-Alsina operators, applying them to select sustainable transportation sharing practices. Gohain, Chutia, and Dutta [15] introduced a symmetric distance within the IF framework, employing it to address pattern recognition and clustering challenges. Ke et al. [16]devised a ranking approach for IFSs, applying it to select sites for photovoltaic poverty alleviation projects. Wan and Yi [17] proposed power average operators for trapezoidal intuitionistic fuzzy numbers, utilizing strict t-norms and t-conorms. The IFS is subject to the constraint that the combined membership and non-membership grades must fall within the range [0, 1]. Nevertheless, Intuitionistic Fuzzy Sets (IFS) prove inadequate when confronted with scenarios where the combined membership and non-membership values (NMV) exceed 1. In response to this limitation, Yager [18] introduced the Pythagorean fuzzy set (PyFS), wherein the total value of the squares of MV and NMV is constrained to 1. Pythagorean fuzzy sets have since been employed to effectively address numerous intricate Multiple Attribute Decision Making (MADM) problems [19, 20]. Further advancements by Yager [21] in 2016 led to the development of the q-rung orthopair fuzzy set (q-ROFS). The use of q-ROFSs proves to be an efficient method for representing ambiguity and uncertainty. Within the framework of q-ROFSs, the combined value of the qth power of both MV and NMV is limited to 1. Wang et al. [22] introduced a q-ROF environment-based MABAC model and utilized it in solving MADM problems. Seikh and Mandal [23] developed q-ROF Frank AOs and utilized them in solving MADM problems. Wang et al. [24] introduced Muirhead mean AOs for fusing q-ROF information. Wang et al. [25] defined q-ROF Hamy mean operators and used them in their work on enterprise resource planning systems. Kausar et al. [26] expanded the CODAS approach to the q-ROF framework and applied it to assess cancer risk. Recent studies have applied various fuzzy set theories across multiple domains. Kabir et al. [27] (2018) used FS in safety and reliability engineering, Mardani et al. [28] (2019) in healthcare and medical problems, and Pinar et al. [29] (2022) in decision-making and classification. IFS were employed by Ngan et al. [30] (2019) for critical decision-making, Jiang et al. [11] for pattern recognition, and Senapati et al. [14] (2023) for global partner selection. PyFS were applied by Haq et al. [31] (2022) in investment policy selection, Dey et al. [32] (2022) in medical diagnosis, and Rani et al. [33](2022) in healthcare waste treatment. In the case of q-ROFS, Kamacı et al. [34, 35] (2022) focused on medical diagnosis and supplier evaluation, while Senapati et al. [36] (2023) applied it to healthcare waste disposal. Recently fuzzy set developed in many form [5, 37, 38]

Aggregation Operators (AOs) simplify data analysis by consolidating information through functions such as sum, average, and count, providing succinct insights. These operators are vital tools for extracting significant patterns and statistics from extensive datasets with ease. In recent years, various aggregation operators have been explored within the q-ROFS environment. Liu et al. [39](2018) introduced Archimedean Bonferroni, while Wei et al. [40] (2018) focused on Heronian mean operators. Garg et al. [41](2020) developed neutrality aggregation operators, and Xing et al. [42] (2020) proposed Hamy mean operators. Further advancements were made by Deveci et al. [43] (2022), who applied Hamacher average and geometric mean operators, and Rawat et al. [44] (2022), who worked on Hamacher Muirhead mean operators. Farid et al. [45] (2023) introduced Aczel–Alsina ordered, hybrid averaging, and geometric mean operators, while Jabeen et al. [46] (2023) developed Aczel-Alsina power Bonferroni operators. Additionally, Qiyas et al. [47] (2023) explored Sine hyperbolic and Dombi operators, and Khan et al. [48] (2023) worked on Aczel–Alsina power averaging and geometric mean operators. These contributions demonstrate the ongoing innovation in aggregation operators for q-ROFS environments, providing valuable tools for solving complex decision-making problems.

This paper introduces novel q-rung orthopair fuzzy ExpoLogarithmic weighted aggregation operators (q-ROFELWAO) and examines their properties. The Expologarithmic t-norm, combining exponential and logarithmic functions, enhances fuzzy logic and decision-making by effectively modeling uncertainty and non-linear relationships. Its ability to capture complex dependencies makes it valuable in AI, control theory, and optimization. To validate the proposed operators, AI is applied within an MCDM framework for sustainable urban innovation and resilience.

1.3 Research gap

Prior to delving into the main contributions of this study, it is crucial to recognize the existing research gaps that provided the impetus for our exploration. The identified gaps include a lack of comprehensive exploration and understanding of ExpoLogarithmic t-norms and t-conorms, particularly in the context of q-rung orthopair fuzzy sets. Furthermore, there has been a dearth of research articulating the fundamental operations of ExpoLogarithmic aggregation tools, hindering the development of a robust theoretical foundation for their practical applications. The absence of specialized aggregation operators tailored for q-rung orthopair fuzzy sets has limited the options available for handling complex fuzzy information. Additionally, the need for a thorough validation of these proposed operators through practical applications, specifically in the realm of MCDM, has been identified. Addressing these gaps forms the basis for the subsequent main contributions of this study.

1.4 Research questions

To guide the investigation and exploration of the proposed study, the following research questions are formulated:

(i) How do ExpoLogarithmic t-norms and t-conorms contribute to the field, and what are their key properties?

(ii) What are the fundamental operations of ExpoLogarithmic aggregation tools within the context of q-rung orthopair fuzzy sets, and how do they provide a foundational understanding for smooth approximation during the aggregation process?

(iii) How do the specialized aggregation operators q-rung orthopair fuzzy ExpoLogarithmic weighted average (q-ROFELWA) and q-rung orthopair fuzzy ExpoLogarithmic weighted geometric (q-ROFELWG) differ from traditional methods, and what are their important properties with illustrative examples?

(iv) How can the proposed approaches be effectively validated through a real-life problem on sustainable urban innovation and resilience using artificial intelligence in MCDM method?

1.5 Issues with earlier works

Several gaps in the existing literature motivate this study. These include the lack of exploration of ExpoLogarithmic t-norms and t-conorms, insufficient clarity on the fundamental operations of ExpoLogarithmic aggregation tools in q-rung orthopair fuzzy sets, and the absence of specialized aggregation operators like the q-ROFELWA and q-ROFELWG. Additionally, there is a lack of robust validation methods, particularly in the context of MCDM, artificial intelligence, and real-life problems related to sustainable urban innovation and resilience. Addressing these gaps is key to the contributions of this study.

1.6 Main contributions of the study

Addressing these research gaps and issues, the main contributions of the proposed study include:

(i) **Introduction of ExpoLogarithmic t-norms and t-conorms:** ExpoLogarithmic t-norm and t-conorm are introduced, and their properties are discussed.

(ii) **Articulation of Fundamental Operations:** The study articulates the essential operations of ExpoLogarithmic aggregation tools within the context of q-rung orthopair fuzzy sets, providing a foundational understanding for achieving a smooth approximation during the aggregation process.

(iii) **Introduction of Specialized Aggregation Operators:** Novel aggregation operators q-rung orthopair fuzzy ExpoLogarithmic weighted average (q-ROFELWA) and q-rung orthopair fuzzy ExpoLogarithmic weighted geometric (q-ROFELWG) are introduced. Their important properties are discussed with examples.

(iv) **Validation through MCDM:** The proposed approaches are rigorously validated through a real-life problem on sustainable urban innovation and resilience by utilizing artificial intelligence in a Multi-Criteria Decision-Making method.

1.7 Significance of the proposed study

The proposed study makes key contributions in several areas. It introduces ExpoLogarithmic t-norms and t-conorms, expanding the fuzzy logic toolkit. It also provides a foundational understanding of ExpoLogarithmic aggregation within q-ROFSs, enabling smoother approximations. The study introduces innovative aggregation operators, like q-ROFELWA and q-ROFELWG, improving aggregation flexibility. Finally, the methods are validated through a real-world application in sustainable urban innovation and resilience, showcasing their effectiveness in complex decision-making. These contributions offer new tools and methodologies for fuzzy set-based decision-making, particularly in Multi-Criteria Decision-Making.

1.8 Organization of the proposed study

The paper is organized as follows: **Section 2 (Methodology)** outlines the core methodological elements crucial for deploying the proposed framework. **Section 3 (Results)** delves into Sustainable Urban Innovation and Resilience utilizing a case study, showcasing the framework's application and presenting numerical results, including the MCDM approach and sensitivity analysis, while comparing with current fuzzy methods to identify their pros and cons. Finally, **Section 4 (Conclusion)** recaps the main findings, considers their implications, addresses the study's limitations, and suggests directions for future research.

2. Methodology

2.1 Preliminaries

This section introduces fundamental concepts in fuzzy set theory and related notions, laying the groundwork for subsequent discussions. Definitions 2.1, 2.2, 2.3, and 2.4 present key concepts such as FS, IFS, PyFS, and q-ROFS. Additionally, Definition 2.5 defines the score function for q-Rung Orthopair Fuzzy Numbers (q-ROFNs), and Definition 2.6 establishes the relationship between the score values of q-ROFNs. Throughout this article, consider $\mathcal{X}$ to be a non-empty set unless explicitly stated otherwise.

Definition 2.1 establishes the foundation for understanding Fuzzy Sets, serving as a fundamental concept for subsequent discussions in this article.

Definition 2.1. [9] The concept of a Fuzzy Set is defined as follows:

$$\mathcal{F} = \{(x, \alpha(x)) : x \in \mathbb{X}\}$$

Here, $\alpha(x)$ is the membership grade of an element $x \in \mathcal{X}$, while $\alpha(x)$ is restricted to values within the interval $[0, 1]$.

Definition 2.2 establishes the groundwork for dealing with Intuitionistic Fuzzy Sets.

Definition 2.2. [10] The concept of an Intuitionistic Fuzzy Set is defined as follows:

$$\mathcal{F} = \{(x, \alpha(x), \beta(x)) : x \in \mathbb{X}\}$$

Here, $\alpha(x)$, $\beta(x)$ are the membership and non-membership grades of an element $x \in \mathcal{X}$, respectively. Both $\alpha(x)$ and $\beta(x)$ are constrained to values within the interval $[0, 1]$, and $\alpha(x) + \beta(x) \leqslant 1$.

Definition 2.3 establishes the groundwork for dealing with Pythagorean Fuzzy Sets.

Definition 2.3. [18] The concept of a Pythagorean Fuzzy Set is defined as follows:

$$\mathcal{F} = \{(x, \alpha(x), \beta(x)) : x \in \mathbb{X}\}$$

Here, $\alpha(x)$, $\beta(x)$ are the membership and non-membership grades of an element $x \in \mathcal{X}$, respectively. Both $\alpha(x)$ and $\beta(x)$ are constrained to values within the interval $[0, 1]$, and $\alpha(x)^2 + \beta(x)^2 \leqslant 1$.

Definition 2.4 establishes the groundwork for dealing with q-rung orthopair fuzzy sets and numbers.

Definition 2.4. [21] The concept of a q-Rung Orthopair Fuzzy Set is defined as follows:

$$\mathcal{F} = \{(x, \alpha(x), \beta(x)) : x \in \mathcal{X}\}$$

Here, $\alpha(x)$ and $\beta(x)$ represent the membership and non-membership grades of an element $x \in \mathcal{X}$, respectively. Both $\alpha(x)$ and $\beta(x)$ are constrained to values within the interval $[0, 1]$, and $\alpha(x)^q + \beta(x)^q \leqslant 1$ $(q \geqslant 1)$. Furthermore, a q-rung Orthopair fuzzy number (q-ROFN) is symbolized as $\mathcal{F} = (\alpha, \beta)$ or $\mathcal{F}_t = (\alpha_t, \beta_t)$ for convenience (where t is a positive integer).

Example 2.1. Real-life Example: Smart Waste Management: Jacintos Nieves et al. [49] highlight the importance of effective waste management for urban sustainability, introducing the m-SWM4Cities model used in Mexico City to assess and improve waste management practices. In a sustainable city, smart waste management systems leverage advanced technologies to optimize waste collection and disposal. The q-ROFS framework can evaluate such systems by addressing uncertainties in waste generation and collection efficiency.

The components of q-ROFS in this context are:

- **Universe of Discourse ($\mathcal{X}$):** The set of all possible waste management scenarios, including varying waste generation levels, weather, and efficiency.
- **Element (x):** A specific scenario, such as a day with variable waste generation and operational factors.
- **Membership Grade ($\alpha(x)$):** The degree to which a scenario belongs to effective waste management, indicating high collection efficiency.
- **Non-membership Grade ($\beta(x)$):** The degree to which a scenario does not belong to effective waste management, indicating inefficiency.

For a given scenario, a q-rung Orthopair fuzzy number (q-ROFN) is defined as:

$$\mathcal{F}_t = (\alpha_t, \beta_t)$$

The constraint $\alpha(x)^q + \beta(x)^q \leqslant 1$ ensures balance between membership and non-membership grades, reflecting uncertainties in waste management. Using the q-ROFS framework, decision-makers can assess smart waste management effectiveness, supporting informed decisions to enhance urban sustainability and resilience.

Liu et al. [50] proposed a score function for any q-ROFN, represented as $\mathcal{F} = (\alpha, \beta)$, defined by $\mathcal{S}(\mathcal{F}) = \alpha^q - \beta^q$. The resulting value of $\mathcal{S}(\mathcal{F})$ may initially fall within the interval $[-1, 1]$. To enhance computational convenience and ensure that $\mathcal{S}(\mathcal{F})$ resides within the more practical interval $[0, 1]$, we have made a slight modification to Liu et al. [50]'s score function for q-ROFN as follows.

Definition 2.5. The score function for any q-ROFN, $\mathcal{F} = (\alpha, \beta)$, is defined as follows:

$$\mathcal{S}(\mathcal{F}) = 1/2 + (1/2)(\alpha^q - \beta^q) \tag{2.1}$$

The connection between the score and q-ROFN as follows.

Definition 2.6. [42] For any two q-ROFNs, $\mathcal{F}_1 = (\alpha_1, \beta_1)$ and $\mathcal{F}_2 = (\alpha_2, \beta_2)$, the relationship between the score values of $\mathcal{F}_1$ and $\mathcal{F}_2$ can be expressed as:

$$\text{If } \mathcal{S}(\mathcal{F}_1) < \mathcal{S}(\mathcal{F}_2) \text{ then } \mathcal{F}_1 < \mathcal{F}_2$$
$$\text{If } \mathcal{S}(\mathcal{F}_1) > \mathcal{S}(\mathcal{F}_2) \text{ then } \mathcal{F}_1 > \mathcal{F}_2$$
$$\text{If } \mathcal{S}(\mathcal{F}_1) = \mathcal{S}(\mathcal{F}_1) \text{ then } \mathcal{F}_1 = \mathcal{F}_2$$

2.2 *ExpoLogarithmic operations on q-Rung orthopair fuzzy numbers*

In this section, we introduce the ExpoLogarithmic t-norm (ELTN) and t-conorm (ELTCN), outlining their key properties. These will serve as foundational tools for the subsequent discussions. We also explore ExpoLogarithmic operations—addition, multiplication, scalar operations, and power—applied to q-rung orthopair fuzzy numbers, using the ELTN (Equation (2.2)) and ELTCN (Equation (2.3)). Illustrative examples and key properties are provided to support the discussion.

Definition 2.7. The ELTN and ELTCN for any two $x, y \in [0, 1]$, and $\mu \in (0, \infty)$ are defined as follows.

$$\text{ELTN}(x, y) = \frac{1}{\mu} \log\left(1 + (e^{\mu x} - 1)(e^{\mu y} - 1)\right) \tag{2.2}$$

$$\text{ELTCN}(x, y) = 1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-x)} - 1)(e^{\mu(1-y)} - 1)) \tag{2.3}$$

The following theorem ensures that the proposed ExpoLogarithmic t-norm (Equation (2.2)) is valid.

Theorem 2.1. Let $T(x, y)$ be the ExpoLogarithmic t-norm as defined in Equation (2.2). It satisfies the following properties:

(i) **Associativity:** For all a, b, c,

$$T(T(a, b), c) = T(a, T(b, c))$$

(ii) **Commutativity:** For all x, y,

$$T(x, y) = T(y, x)$$

(iii) **Non-decreasing:** For all a, x, y such that $x \leqslant y$,

$$x \leqslant y \implies T(a,x) \leqslant T(a,y) \quad \text{and} \quad x \leqslant y \implies T(x,a) \leqslant T(y,a)$$

(iv) **Boundedness:** For all x, y, $0 \leqslant T(x,y) \leqslant 1$

In the following Definition 2.8, we define ExpoLogarithmic Operations, $\oplus$ and $\otimes$ between two q-ROFNs.

Definition 2.8. The ExpoLogarithmic Operations, $\oplus$ and $\otimes$ between two q-ROFNs, $\mathcal{F}_1 = (\alpha_1, \beta_1)$ and $\mathcal{F}_2 = (\alpha_2, \beta_2)$, with $\mu, \delta > 0$, are defined as follows:

$$\mathcal{F}_1 \oplus \mathcal{F}_2 = \left(\left[1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-\alpha_1^q)} - 1)(e^{\mu(1-\alpha_2^q)} - 1)) \right]^{1/q}, \left[\frac{1}{\mu} \log(1 + (e^{\mu\beta_1^q} - 1)(e^{\mu\beta_2^q} - 1)) \right]^{1/q} \right) \tag{2.4}$$

$$\mathcal{F}_1 \otimes \mathcal{F}_2 = \left(\left[\frac{1}{\mu} \log(1 + (e^{\mu\alpha_1^q} - 1)(e^{\mu\alpha_2^q} - 1)) \right]^{1/q}, \left[1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-\beta_1^q)} - 1)(e^{\mu(1-\beta_2^q)} - 1)) \right]^{1/q} \right) \tag{2.5}$$

$$\delta\mathcal{F}_1 = \left(\left[1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-\alpha_1^q)} - 1)^{\delta}) \right]^{1/q}, \left[\frac{1}{\mu} \log(1 + (e^{\mu\beta_1^q} - 1)^{\delta}) \right]^{1/q} \right) \tag{2.6}$$

$$\mathcal{F}_1^{\delta} = \left(\left[\frac{1}{\mu} \log(1 + (e^{\mu\alpha_1^q} - 1)^{\delta}) \right]^{1/q}, \left[1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-\beta_1^q)} - 1)^{\delta}) \right]^{1/q} \right) \tag{2.7}$$

Consider the essential properties of the ExpoLogarithmic operations $\oplus$ and $\otimes$, as well as scalar multiplication and scalar power, as outlined in Theorem 2.2.

Theorem 2.2. Let $\mathcal{F}_1$ and $\mathcal{F}_2$ represent two q-ROFNs, $\mathcal{F}_1 = (\alpha_1, \beta_1)$ and $\mathcal{F}_2 = (\alpha_2, \beta_2)$. For any $\delta_1, \delta_2 > 0$, the following holds.

(1) $\mathcal{F}_1 \oplus \mathcal{F}_2 = \mathcal{F}_2 \oplus \mathcal{F}_1$.

(2) $\mathcal{F}_1 \otimes \mathcal{F}_2 = \mathcal{F}_2 \otimes \mathcal{F}_1$.

(3) $\delta_1(\mathcal{F}_1 \oplus \mathcal{F}_2) = \delta_1\mathcal{F}_1 \oplus \delta_1\mathcal{F}_2$.

(4) $(\delta_1 + \delta_2)\mathcal{F}_1 = \delta_1\mathcal{F}_1 \oplus \delta_2\mathcal{F}_1$.

(5) $(\mathcal{F}_1 \otimes \mathcal{F}_2)^{\delta_1} = \mathcal{F}_1^{\delta_1} \otimes \mathcal{F}_2^{\delta_1}$.

(6) $\mathcal{F}_1^{\delta_1} \otimes \mathcal{F}_1^{\delta_2} = \mathcal{F}_1^{\delta_1+\delta_2}$.

Proof. The proof is easily shown by applying Definition 2.8. $\square$

2.3 *Novel q-Rung orthopair fuzzy expoLogarithmic aggregation operators and their unique characteristics*

This section introduces two groundbreaking aggregation operators, namely q-ROFELWA and q-ROFELWG, designed for combining q-rung orthopair fuzzy numbers (q-ROFNs). Through illustrative examples, we investigate their properties, including idempotency, boundedness, and monotonicity. The exploration begins with the introduction of q-ROFELWA in Subsection 2.3.1.

2.3.1 *Exploring the q-ROFELWA operator and its fundamental Characteristics*

In this section, we present the q-ROFELWA aggregation operator, illustrating its capability to seamlessly integrate q-ROFNs within the framework of q-ROFSs. We offer a lucid demonstration through a straightforward example and thoroughly explore the essential properties of the operator. The q-ROFELWA operator is formally defined in Definition 2.9, utilizing the ExpoLogarithmic operation $\oplus$ (Equation (2.4)) and scalar multiplication (Equation (2.6)).

Definition 2.9. Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$ denote q-rung orthopair fuzzy numbers (q-ROFNs). Consider a weight vector $\eta = [\eta_1, \eta_2, \ldots, \eta_m]$ associated with the q-ROFNs $\mathcal{F}_i$ such that $\eta_i > 0$ and $\sum_{i=1}^{m} \eta_i = 1$. The q-ROFELWA operator is then defined as

$$\text{q-ROFELWA}(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i.$$

The subsequent Theorem 2.3 establishes the aggregated value of q-rung orthopair fuzzy numbers (q-ROFNs), denoted as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, when the q-ROFELWA operator is applied.

Theorem 2.3. The aggregated value of q-rung orthopair fuzzy numbers (q-ROFNs), denoted as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, upon applying the q-ROFELWA operator, remains a q-ROFN. The aggregation is computed as follows:

$$q - ROEFLWA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m} \eta_i \mathcal{F}_i$$

$$= \left(\left[1 - \frac{1}{\mu} \log(1 + \prod_{i=1}^{m} (e^{\mu(1-\alpha_i^q)} - 1)^{\eta_i}) \right]^{1/q}, \left[\frac{1}{\mu} \log(1 + \prod_{i=1}^{m} (e^{\mu \beta_i^q} - 1)^{\eta_i}) \right]^{1/q} \right) \tag{2.8}$$

Proof. Mathematical induction offers a dependable approach for verifying the validity of a proof.

Case 1. When $m = 2$, the condition $\sum_{i=1}^{2} \eta_i = 1$ is satisfied, leading to:

$$q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2) = \oplus_{i=1}^{2} \eta_i \mathcal{F}_i$$

$$= \left(\left[1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-\alpha_1^q)} - 1)^{\eta_1} (e^{\mu(1-\alpha_2^q)} - 1)^{\eta_2}) \right]^{1/q}, \left[\frac{1}{\mu} \log(1 + (e^{\mu \beta_1^q} - 1)^{\eta_1} (e^{\mu \beta_2^q} - 1)^{\eta_2}) \right]^{1/q} \right)$$
$$= \left(\left[1 - \frac{1}{\mu} \log(1 + \prod_{i=1}^{2} (e^{\mu(1-\alpha_i^q)} - 1)^{\eta_i}) \right]^{1/q}, \left[\frac{1}{\mu} \log(1 + \prod_{i=1}^{2} (e^{\mu \beta_i^q} - 1)^{\eta_i}) \right]^{1/q} \right)$$

Case 2. Assuming the result holds true for $m = p$, if $m = p + 1$, then $\sum_{i=1}^{p+1} \eta_i = 1$, and we obtain

$$q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2, \ldots \mathcal{F}_p, \mathcal{F}_{p+1}) = \oplus_{i=1}^{p} \eta_i \mathcal{F}_i \oplus \eta_{p+1} \mathcal{F}_{p+1}$$

$$= \left(\left[1-\frac{1}{\mu}\log(1+\prod_{i=1}^{p}(e^{\mu(1-\alpha_i^q)}-1)^{\eta_i}(e^{\mu(1-\alpha_{p+1}^q)}-1)^{\eta_{p+1}})\right]^{1/q},\right.$$
$$\left.\left[\frac{1}{\mu}\log(1+\prod_{i=1}^{p}(e^{\mu\beta_i^q}-1)^{\eta_i}(e^{\mu\beta_{p+1}^q}-1)^{\eta_{p+1}})\right]^{1/q}\right)$$
$$= \left(\left[1-\frac{1}{\mu}\log(1+\prod_{i=1}^{p+1}(e^{\mu(1-\alpha_i^q)}-1)^{\eta_i})\right]^{1/q}, \left[\frac{1}{\mu}\log(1+\prod_{i=1}^{p+1}(e^{\mu\beta_i^q}-1)^{\eta_i})\right]^{1/q}\right)$$

Hence, the obtained result is valid for all values of m.

□

To illustrate Theorem 2.3, we focus on Example 2.2, where we demonstrate the computation of aggregation values for two q-ROFNs using the q-ROFELWA operator (Equation (2.8)).

Example 2.2. Consider two q-ROFNs, denoted as $\mathcal{F}_1 = (0.8, 0.4)$ and $\mathcal{F}_2 = (0.9, 0.7)$, where the parameters are set to $\mu = 2$ and $q = 4$. Additionally, we have the weight vector $\eta = [0.6, 0.4]$. Now, we can determine the aggregation value of $\mathcal{F}_1$ and $\mathcal{F}_2$ using the q-ROFELWA aggregation operator, as defined in Equation (2.8).

$$q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2) = \oplus_{i=1}^{2}\eta_i\mathcal{F}_i \quad = (\alpha, \beta) \tag{2.9}$$

where α is computed by utilizing Equation (2.8) and given data as follows.

$$\alpha = \left[1-\frac{1}{\mu}\log(1+\prod_{i=1}^{m}(e^{\mu(1-\alpha_i^q)}-1)^{\eta_i})\right]^{1/q}$$
$$= \left[1-\frac{1}{2}\log(1+(e^{2(1-0.8^4)}-1)^{0.6}(e^{2(1-0.9^4)}-1)^{0.4})\right]^{1/4}$$
$$= \left[1-\frac{1}{2}\log(1+(1.629733082)(0.995719943))\right]^{1/4}$$
$$= 0.848318069$$

Likewise, we can calculate the values of β in Equation (2.9) by employing Equation (2.8) alongside the provided data. The resultant aggregation value is presented below:

$$q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2) = (0.848318069, 0.5064895)$$

The Idempotency Property of the q-ROFELWA aggregation operator within q-ROFSs is explored in Theorem 2.4.

Theorem 2.4. (Idempotency Property) If $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots m$ such that $\mathcal{F}_i = \mathcal{F}$, where $\mathcal{F} = (\alpha, \beta)$, then $q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \mathcal{F}$.

Proof. Since $\mathfrak{R}_i = \mathfrak{R}$ for all $\mathfrak{R} = \alpha, \beta$, Equation (2.8) transforms to:

$$q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \oplus_{i=1}^{m}\eta_i\mathcal{F}_i$$

$$= \left(\left[1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-\alpha^q)} - 1)^{\sum_{i=1}^m \eta_i}) \right]^{1/q}, \left[\frac{1}{\mu} \log(1 + (e^{\mu\beta^q} - 1)^{\sum_{i=1}^m \eta_i}) \right]^{1/q} \right) \tag{2.10}$$

Due to the condition $\sum_{i=1}^m \eta_i = 1$, the expression in Equation (2.10) transforms to (α, β). Thus,

$$q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2, \ldots \mathcal{F}_m) = \mathcal{F}.$$

□

In Theorem 2.5, we examine the Boundedness Property of the proposed aggregation operator, q-ROFELWA, within the context of q-ROFSs.

Theorem 2.5. (Boundedness Property) Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots m$ be the q-ROFNs. Assume that

$$\mathcal{F}^- = (\min_i\{\alpha_i\}, \max_i\{\beta_i\}),$$
$$\mathcal{F}^+ = (\max_i\{\alpha_i\}, \min_i\{\beta_i\})$$

We establish the inequality $\mathcal{F}^- \leqslant$ q-ROFELWA$(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leqslant \mathcal{F}^+$.

Proof. For any index $i = 1, 2, \ldots, m$, the following inequalities hold:

$$\min_i \alpha_i \leqslant \alpha_i \leqslant \max_i \alpha_i,$$
$$\max_i \beta_i \geqslant \beta_i \geqslant \min_i \beta_i.$$

Consequently, we obtain:

$$\min_i \alpha_i \leqslant \left[1 - \tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^m (e^{\mu(1-\alpha_i^q)} - 1)^{\eta_i}) \right]^{1/q} \leqslant \max_i \alpha_i,$$

$$\max_i \beta_i \geqslant \left[\tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^m (e^{\mu\beta_i^q} - 1)^{\eta_i}) \right]^{1/q} \geqslant \min_i \beta_i.$$

It implies that

$$\mathcal{S}(\mathcal{F}^-) \leqslant \mathcal{S}(q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m)) \leqslant \mathcal{S}(\mathcal{F}^+)$$

by Equation (2.1) and Equation (2.8). Thus, the proof is concluded by Definition 2.6.

□

In Theorem 2.6, we explore the Monotonicity Property of the proposed aggregation operator, q-ROFELWA, within the framework of q-ROFSs.

Theorem 2.6. (Monotonicity Property) Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots m$ and $\mathcal{F}_i^{'} = (\alpha_i^{'}, \beta_i^{'})$ for $i = 1, 2, \ldots m$ be the q-ROFNs. If $\mathcal{F}_i \leqslant \mathcal{F}_i^{'}$ for all $i = 1, 2, \ldots, m$, then $q - ROFELWA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leqslant q - ROFELWA(\mathcal{F}_1^{'}, \mathcal{F}_2^{'}, \ldots, \mathcal{F}_m^{'})$.

Proof. Assuming $\mathcal{F}_i \leqslant \mathcal{F}_i^{'}$ for all $i = 1, 2, \ldots, m$, as per Definition 2.6, we have

$$\mathcal{S}(\mathcal{F}_i) \leqslant \mathcal{S}(\mathcal{F}_i^{'}) \quad \text{for} \quad i = 1, 2, \ldots, m.$$

Using this, we establish the inequalities, $\alpha_i \leqslant \alpha_i^{'}$ and $\beta_i \geqslant \beta_i^{'}$. Consequently, we obtain:

$$\left[1 - \tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^m (e^{\mu(1-\alpha_i^q)} - 1)^{\eta_i}) \right]^{1/q} \leqslant \left[1 - \tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^m (e^{\mu(1-\alpha_i^{q\,'})} - 1)^{\eta_i}) \right]^{1/q},$$

$$\left[\frac{1}{\mu}\log(1+\prod_{i=1}^{m}(e^{\mu\beta_i^q}-1)^{\eta_i})\right]^{1/q} \geqslant \left[\frac{1}{\mu}\log(1+\prod_{i=1}^{m}(e^{\mu\beta_i^{q'}}-1)^{\eta_i})\right]^{1/q}.$$

This implies that

$$\mathcal{S}(q-ROFELWA(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m)) \leqslant \mathcal{S}(q-ROFELWA(\mathcal{F}_1^{'},\mathcal{F}_2^{'},\ldots\mathcal{F}_m^{'}))$$

by Equation (2.1) and Equation (2.8). Thus, the proof is concluded by Definition 2.6. □

2.3.2 *Exploring the q-ROFELWG operator and its fundamental characteristics*

In this section, we present the q-ROFELWG aggregation operator, illustrating its capability to seamlessly integrate q-ROFNs within the framework of q-ROFSs. We offer a lucid demonstration through a straightforward example and thoroughly explore the essential properties of the operator. The q-ROFELWG operator is formally defined in Definition 2.10, utilizing the ExpoLogarithmic operation $\otimes$ (Equation (2.5)) and scalar power (Equation (2.7)).

Definition 2.10. Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$ denote q-rung orthopair fuzzy numbers (q-ROFNs). Consider a weight vector $\eta = [\eta_1, \eta_2, \ldots, \eta_m]$ associated with the q-ROFNs $\mathcal{F}_i$ such that $\eta_i > 0$ and $\sum_{i=1}^{m}\eta_i = 1$. The q-ROFELWG operator is then defined as

$$\text{q-ROFELWG}(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m) = \otimes_{i=1}^{m}\mathcal{F}_i{}^{\eta_i}.$$

The subsequent Theorem 2.7 establishes the aggregated value of q-rung orthopair fuzzy numbers (q-ROFNs), denoted as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, when the q-ROFELWG operator is applied.

Theorem 2.7. The aggregated value of q-rung orthopair fuzzy numbers (q-ROFNs), denoted as $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots, m$, upon applying the q-ROFELWG operator, remains a q-ROFN. The aggregation is computed as follows:

$$q-ROFELWG(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m) = \otimes_{i=1}^{m}\mathcal{F}_i^{\eta_i}$$

$$= \left(\left[\frac{1}{\mu}\log(1+\prod_{i=1}^{m}(e^{\mu\alpha_i^q}-1)^{\eta_i})\right]^{1/q}, \left[1-\frac{1}{\mu}\log(1+\prod_{i=1}^{m}(e^{\mu(1-\beta_i^q)}-1)^{\eta_i})\right]^{1/q}\right) \tag{2.11}$$

Proof. Mathematical induction offers a dependable approach for verifying the validity of a proof.

Case 1. When $m = 2$, the condition $\sum_{i=1}^{2}\eta_i = 1$ is satisfied, leading to:

$$q-ROFELWG(\mathcal{F}_1,\mathcal{F}_2) = \otimes_{i=1}^{2}\mathcal{F}_i^{\eta_i}$$

$$= \left(\left[\frac{1}{\mu}\log(1+(e^{\mu\alpha_1^q}-1)^{\eta_1}(e^{\mu\alpha_2^q}-1)^{\eta_2})\right]^{1/q}, \left[1-\frac{1}{\mu}\log(1+(e^{\mu(1-\beta_1^q)}-1)^{\eta_1}(e^{\mu(1-\beta_2^q)}-1)^{\eta_2})\right]^{1/q}\right)$$
$$= \left(\left[\frac{1}{\mu}\log(1+\prod_{i=1}^{2}(e^{\mu\alpha_i^q}-1)^{\eta_i})\right]^{1/q}, \left[1-\frac{1}{\mu}\log(1+\prod_{i=1}^{2}(e^{\mu(1-\beta_i^q)}-1)^{\eta_i})\right]^{1/q}\right)$$

Case 2. Assuming the result holds true for $m = p$, if $m = p + 1$, then $\sum_{i=1}^{p+1}\eta_i = 1$, and we obtain

$$q-ROFELWG(\mathcal{F}_1,\mathcal{F}_2,\ldots\mathcal{F}_p,\mathcal{F}_{p+1}) = \otimes_{i=1}^{p}\mathcal{F}_i^{\eta_i} \otimes \mathcal{F}_{p+1}^{\eta_{p+1}}$$

$$= \left(\left[\frac{1}{\mu} \log(1 + \prod_{i=1}^{p} (e^{\mu\alpha_i^q} - 1)^{\eta_i} (e^{\mu\alpha_{p+1}^q} - 1)^{\eta_{p+1}}) \right]^{1/q} \right.$$
$$\left. \left[1 - \frac{1}{\mu} \log(1 + \prod_{i=1}^{p} (e^{\mu(1-\beta_i^q)} - 1)^{\eta_i} (e^{\mu(1-\beta_{p+1}^q)} - 1)^{\eta_{p+1}}) \right]^{1/q} \right)$$
$$= \left(\left[\frac{1}{\mu} \log(1 + \prod_{i=1}^{p+1} (e^{\mu\alpha_i^q} - 1)^{\eta_i}) \right]^{1/q} \left[1 - \frac{1}{\mu} \log(1 + \prod_{i=1}^{p+1} (e^{\mu(1-\beta_i^q)} - 1)^{\eta_i}) \right]^{1/q} \right)$$

Hence, the obtained result is valid for all values of m.

□

To illustrate Theorem 2.7 , we focus on Example 2.3, where we demonstrate the computation of aggregation values for two q-ROFNs using the q-ROFELWG operator (Equation (2.11)).

Example 2.3. Consider two q-ROFNs, denoted as $\mathcal{F}_1 = (0.8, 0.4)$ and $\mathcal{F}_2 = (0.9, 0.7)$, where the parameters are set to $\mu = 2$ and $q = 4$. Additionally, we have the weight vector $\eta = [0.6, 0.4]$. Now, we can determine the aggregation value of $\mathcal{F}_1$ and $\mathcal{F}_2$ using the q-ROFELWG aggregation operator, as defined in Equation (2.12).

$$q - ROFELWG(\mathcal{F}_1, \mathcal{F}_2) = \otimes_{i=1}^{2} \mathcal{F}_i^{\eta_i} = (\alpha, \beta) \tag{2.12}$$

where α is computed by utilizing Equation (2.11) and given data as follows.

$$\begin{aligned} \alpha &= \left[\frac{1}{\mu} \log(1 + \prod_{i=1}^{m} (e^{\mu\alpha_i^q} - 1)^{\eta_i}) \right]^{1/q} \\ &= \left[\frac{1}{2} \log(1 + (e^{2(0.8^4)} - 1)^{0.6} (e^{2(0.9^4)} - 1)^{0.4}) \right]^{1/4} \\ &= \left[\frac{1}{2} \log(1 + (1.153485425)(1.490958173)) \right]^{1/4} \\ &= 0.841013655 \end{aligned}$$

Likewise, we can calculate the values of β in Equation (2.12) by employing Equation (2.11) alongside the provided data. The resultant aggregation value is presented below:

$$q - ROFELWG(\mathcal{F}_1, \mathcal{F}_2) = (0.841013655, 0.580764307)$$

The Idempotency Property of the q-ROFELWG aggregation operator within q-ROFSs is explored in Theorem 2.8.

Theorem 2.8. (Idempotency Property) If $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots m$ such that $\mathcal{F}_i = \mathcal{F}$, where $\mathcal{F} = (\alpha, \beta)$, then $q - ROFELWG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \mathcal{F}$.

Proof. Since $\mathfrak{R}_i = \mathfrak{R}$ for all $\mathfrak{R} = \alpha, \beta$, Equation (2.11) transforms to:

$$q - ROFELWG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) = \otimes_{i=1}^{m} \mathcal{F}_i^{\eta_i}$$

$$= \left(\left[\frac{1}{\mu} \log(1 + (e^{\mu \alpha^q} - 1)^{\sum_{i=1}^{m} \eta_i}) \right]^{1/q}, \left[1 - \frac{1}{\mu} \log(1 + (e^{\mu(1-\beta^q)} - 1)^{\sum_{i=1}^{m} \eta_i}) \right]^{1/q} \right) \tag{2.13}$$

Due to the condition $\sum_{i=1}^{m} \eta_i = 1$, the expression in Equation (2.13) transforms to (α, β). Thus,

$$q - ROFELWG(\mathcal{F}_1, \mathcal{F}_2, \ldots \mathcal{F}_m) = \mathcal{F}.$$

□

In Theorem 2.9, we examine the Boundedness Property of the proposed aggregation operator, q-ROFELWG, within the context of q-ROFSs.

Theorem 2.9. (Boundedness Property) Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots m$ be the q-ROFNs. Assume that

$$\mathcal{F}^- = (\min_i\{\alpha_i\}, \max_i\{\beta_i\}),$$
$$\mathcal{F}^+ = (\max_i\{\alpha_i\}, \min_i\{\beta_i\})$$

We establish the inequality $\mathcal{F}^- \leqslant$ q-ROFELWG$(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leqslant \mathcal{F}^+$.

Proof. For any index $i = 1, 2, \ldots, m$, the following inequalities hold:

$$\min_i \alpha_i \leqslant \alpha_i \leqslant \max_i \alpha_i,$$
$$\max_i \beta_i \geqslant \beta_i \geqslant \min_i \beta_i.$$

Consequently, we obtain:

$$\min_i \alpha_i \leqslant \left[\tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^{m} (e^{\mu \alpha_i^q} - 1)^{\eta_i}) \right]^{1/q} \leqslant \max_i \alpha_i,$$

$$\max_i \beta_i \geqslant \left[1 - \tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^{m} (e^{\mu(1-\alpha_i^q)} - 1)^{\eta_i}) \right]^{1/q} \geqslant \min_i \beta_i.$$

It implies that

$$\mathcal{S}(\mathcal{F}^-) \leqslant \mathcal{S}(q - ROFELWG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m)) \leqslant \mathcal{S}(\mathcal{F}^+)$$

by Equation (2.1) and Equation (2.11). Thus, the proof is concluded by Definition 2.6.

□

In Theorem 2.10, we explore the Monotonicity Property of the proposed aggregation operator, q-ROFELWG, within the framework of q-ROFSs.

Theorem 2.10. (Monotonicity Property) Let $\mathcal{F}_i = (\alpha_i, \beta_i)$ for $i = 1, 2, \ldots m$ and $\mathcal{F}_i^{'} = (\alpha_i^{'}, \beta_i^{'})$ for $i = 1, 2, \ldots m$ be the q-ROFNs. If $\mathcal{F}_i \leqslant \mathcal{F}_i^{'}$ for all $i = 1, 2, \ldots, m$, then $q - ROFELWG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_m) \leqslant q - ROFELWG(\mathcal{F}_1^{'}, \mathcal{F}_2^{'}, \ldots, \mathcal{F}_m^{'})$.

Proof. Assuming $\mathcal{F}_i \leqslant \mathcal{F}_i^{'}$ for all $i = 1, 2, \ldots, m$, as per Definition 2.6, we have

$$\mathcal{S}(\mathcal{F}_i) \leqslant \mathcal{S}(\mathcal{F}_i^{'}) \quad \text{for} \quad i = 1, 2, \ldots, m.$$

Using this, we establish the inequalities, $\alpha_i \leqslant \alpha_i^{'}$ and $\beta_i \geqslant \beta_i^{'}$. Consequently, we obtain:

$$\left[\tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^{m} (e^{\mu \alpha_i^q} - 1)^{\eta_i}) \right]^{1/q} \leqslant \left[\tfrac{1}{\mu} \log(1 + \textstyle\prod_{i=1}^{m} (e^{\mu \alpha_i^{q'}} - 1)^{\eta_i}) \right]^{1/q},$$

$$\left[1-\frac{1}{\mu}\log(1+\prod_{i=1}^{m}(e^{\mu(1-\beta_i^q)}-1)^{\eta_i})\right]^{1/q} \geqslant \left[1-\frac{1}{\mu}\log(1+\prod_{i=1}^{m}(e^{\mu(1-\beta_i^{q\prime})}-1)^{\eta_i})\right]^{1/q}.$$

This implies that

$$\mathcal{S}(q-\mathrm{ROFELWG}(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_m)) \leqslant \mathcal{S}(q-\mathrm{ROFELWG}(\mathcal{F}_1',\mathcal{F}_2',\ldots\mathcal{F}_m'))$$

by Equation (2.1) and Equation (2.8). Thus, the proof is concluded by Definition 2.6. □

2.4 *Methodology for multi-criteria decision making and its practical application*

In the domain of decision-making, where a collection of alternatives $\mathbb{S} = \{\mathbb{S}_1, \mathbb{S}_2, \ldots, \mathbb{S}_n\}$ undergoes evaluation against a set of criteria $\eta = \{\eta_1, \eta_2, \ldots, \eta_m\}$, the evaluation of alternative $\mathbb{S}_j$ $(j = 1, 2, \ldots, n)$ under the criteria η_i $(i = 1, 2, \ldots, m)$ is encapsulated in the tabular representation of q-rung orthopair fuzzy numbers, as depicted in Table 1.

Table 1
Tabulated Representation of q-rung orthopair fuzzy numbers

$(\eta/\mathbb{S})$	$\mathbb{S}_1$	$\mathbb{S}_2$	$\ldots$	$\mathbb{S}_n$
η_1	$(\alpha_{11}, \beta_{11})$	$(\alpha_{12}, \beta_{12})$	$\ldots$	$(\alpha_{1n}, \beta_{1n})$
η_2	$(\alpha_{21}, \beta_{21})$	$(\alpha_{22}, \beta_{22})$	$\ldots$	$(\alpha_{2n}, \beta_{2n})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
η_m	$(\alpha_{m1}, \beta_{m1})$	$(\alpha_{m2}, \beta_{m2})$	$\ldots$	$(\alpha_{mn}, \beta_{mn})$

Subsequently, we introduce a Multi-Criteria Decision-Making method (MCDM) delineated through the following steps, as illustrated in the flowchart presented in Figure 1.

Step 1: Combine all criteria values from the tabular representation of q-rung orthopair fuzzy numbers, as demonstrated in Table 1, utilizing either the q-ROFELWA operator (Equation (2.8)) or the q-ROFELWG operator (Equation (2.11)).

Step 2: Compute the combined criteria values' score using Equation (2.1).

Step 3: Determine the ranking order based on the acquired score values.

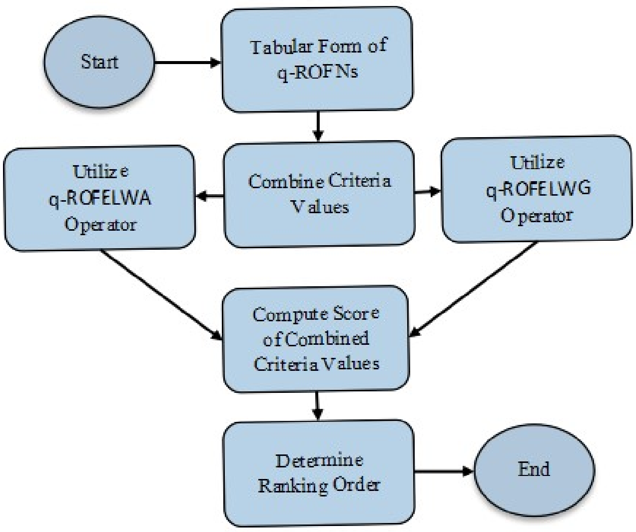

Fig. 1. MCDM Flowchart

3. Results

3.1 Case study

In the domain of urban planning and sustainability, Urban Planners (UPs) are tasked with evaluating alternative approaches to enhance city resilience and reduce environmental impact. Consider four sustainability proposals, denoted as $\mathbb{S}_1$, $\mathbb{S}_2$, $\mathbb{S}_3$, and $\mathbb{S}_4$, utilizing cutting-edge technologies, including Artificial Intelligence (AI). UPs consider the criteria of Environmental Impact (Θ_1), Economic Viability (Θ_2), Community Inclusivity (Θ_3), and Technological Feasibility (Θ_4) for evaluating sustainability alternatives, as outlined below:

(i) **Smart Green Buildings ($\mathbb{S}_1$):** The environmental impact of smart green buildings is significant, as they incorporate advanced technologies, including Artificial Intelligence, to optimize energy efficiency and reduce waste. Equally important is the evaluation of their economic viability, which involves assessing the costs of construction and maintenance while accounting for long-term sustainability gains through AI-driven solutions. Community inclusivity plays a critical role in these developments, as smart green buildings foster an inclusive urban environment that promotes accessibility and affordability for diverse populations. Additionally, technological feasibility is essential, involving the integration and management of advanced systems like AI within urban infrastructure to support these forward-thinking initiatives

(ii) **Urban Green Spaces and Biodiversity Corridors ($\mathbb{S}_2$):** The development of urban green spaces positively impacts the environment by fostering biodiversity corridors and green areas, supported by advanced ecological monitoring technologies, including AI. Economic viability is a crucial consideration, involving the evaluation of establishment and maintenance costs while balancing these with the long-term ecological benefits enabled by AI-driven strategies. Equally important is community inclusivity, as these green spaces, guided by AI, promote engagement and inclusivity by ensuring accessibility and cultural relevance for diverse populations. Furthermore, technological feasibility must be assessed, focusing on the integration and management of advanced ecological monitoring systems, including AI, to support the effective development of urban green spaces.

(iii) **Renewable Energy Microgrids ($\mathbb{S}_3$):** Decentralized renewable energy microgrids offer significant environmental benefits by integrating solar, wind, and other sustainable energy sources, optimized through AI technologies. The economic viability of these systems requires careful evaluation, focusing on the cost-effectiveness of their implementation and maintenance while utilizing AI-based decision support. Community inclusivity is a vital aspect, as AI-guided microgrids contribute to enhanced energy accessibility and affordability, particularly for underserved populations. Additionally, technological feasibility plays a key role, involving the integration and management of diverse renewable energy sources within urban microgrid systems through advanced AI solutions.

(iv) **Smart Urban Mobility Solutions ($\mathbb{S}_4$):**

Smart and sustainable urban mobility solutions offer notable environmental benefits by leveraging advanced technologies, including AI-driven optimizations, to enhance transportation efficiency and reduce emissions. Economic viability is a critical consideration, involving the assessment of implementation and operational costs while balancing these with long-term environmental gains through AI-informed strategies. Community inclusivity is central to these initiatives, as AI-enhanced mobility solutions promote accessibility for all members of society, helping to reduce transportation disparities and foster equitable access. Moreover, technological feasibility plays a crucial role, focusing on the integration of advanced transportation technologies within urban infrastructure to support the effective deployment of smart urban mobility systems.

3.2 Numerical results

This comprehensive problem is tackled through the proposed Multi-Criteria Decision-Making (MCDM) approach. The tabular presentation in Table 2 illustrates the evaluation scores for alternatives $\mathbb{S}_1$, $\mathbb{S}_2$, $\mathbb{S}_3$, and $\mathbb{S}_4$ concerning criteria η_1, η_2, η_3, and η_4 within the framework of q-ROFNs. Let $\omega = [0.2, 0.3, 0.1, 0.4]$ represent a weight vector for the criteria.

Table 2
Tabular form of q-ROFNs (q=3)

$(\eta/\mathbb{S})$	$\mathbb{S}_1$	$\mathbb{S}_2$	$\mathbb{S}_3$	$\mathbb{S}_4$
η_1	(0.9,0.4)	(0.2,0.6)	(0.8,0.1)	(0.3,0.5)
η_2	(0.9,0.3)	(0.4,0.8)	(0.4,0.2)	(0.2,0.2)
η_3	(0.9,0.5)	(0.5,0.9)	(0.7,0.3)	(0.3,0.5)
η_4	(0.8,0.2)	(0.3,0.6)	(0.7,0.4)	(0.3,0.4)

Utilizing the proposed MCDM approach, we outline the decision steps as follows:

Step 1: Combine all criteria values from the tabular representation of q-rung orthopair fuzzy numbers (Table 2), utilizing either the q-ROFELWA operator (Equation (2.8)) or the q-ROFELWG operator (Equation (2.11)). The tabulated values representing the combined criteria for q-rung orthopair fuzzy numbers (q-ROFNs) under the operators q-ROFELWA and q-ROFELWG (with $q = 3$ and $\mu = 2$) are presented in Table 3.

Table 3
Combined Criteria Values for q-ROFNs (Table 2) with Respect to Proposed Operators

Operator	$\mathbb{S}_1$	$\mathbb{S}_2$	$\mathbb{S}_3$	$\mathbb{S}_4$
q-ROFELWA	(0.8673521, 0.2856485)	(0.3538543 , 0.689544)	(0.6706149, 0.2406238)	(0.2772537, 0.3501081)
q-ROFELWG	(0.8611213 , 0.3164538)	(0.3234156 , 0.7054711)	(0.6349121, 0.2951396)	(0.2670195 , 0.3888973)

Step 2: Compute the combined criteria values' score using Equation (2.1). The calculated score values are presented in Table 4.

Table 4
Performance Scores for Alternative Operators

Operator	$\mathbb{S}_1$	$\mathbb{S}_2$	$\mathbb{S}_3$	$\mathbb{S}_4$
q-ROFELWA	0.8146006	0.3582245	0.6438299	0.4891988
q-ROFELWG	0.8206934	0.3034541	0.658003	0.4600292

Step 3: Determine the ranking order based on the acquired score values (Table 4). The tabulated ranking order for operators q-ROFELWA and q-ROFELWG (with $q = 3$ and $\mu = 2$) is illustrated in Table 5.

Table 5
Operators Ranking

Operator	**Ranking**
q-ROFELWA	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFELWG	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$

Based on the outcomes detailed in Table 5, the preference ranking $\mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_1 \succ \mathbb{S}_2$ is established. As a result, $\mathbb{S}_3$ stands out as a preferred option for fortifying city resilience and mitigating environmental impact within the devised Multiple Criteria Decision Making (MCDM) framework.

3.3 *Exploring sensitivity: q-ROFELWA and q-ROFELWG aggregation operators*

Table 6 provides an in-depth exploration of the sensitivity of the proposed aggregation operators, q-ROFELWA and q-ROFELWG, across various q and μ values. For $q = 3$ and $\mu = 5$, q-ROFELWA demonstrates significant performance with score values $\mathbb{S}_1 = 0.8119127$ and $\mathbb{S}_3 = 0.6404214$, ranking highest among the considered operators. Similarly, q-ROFELWG follows closely with a commendable performance, showcasing the robustness of these operators in this setting. As the values of q and μ change, the proposed operators consistently maintain their effectiveness. Notably, for $q = 30$ and $\mu = 2$, q-ROFELWA and q-ROFELWG exhibit balanced score values across all criteria, emphasizing their stability and versatility. In the case of $q = 20$ and $q = 30$, the operators maintain their superiority, with q-ROFELWA and q-ROFELWG consistently outperforming other operators across different μ values. The rankings clearly indicate the strength of these proposed aggregation operators in handling variations in both q and μ. In conclusion, the q-ROFELWA and q-ROFELWG aggregation operators show remarkable sensitivity and robustness, making them valuable choices for diverse scenarios and reinforcing their suitability for complex decision-making processes. Figure 2 presents the rankings when q is fixed and μ varies for the proposed operators. In a similar manner, Figure 3 shows the rankings with μ fixed and q varying for the proposed operators.

Table 6
Score Values and Rankings of Proposed Operators across Different q and μ Values

q	μ	**Proposed Operator**	**Score Values** $\mathbb{S}_1$	$\mathbb{S}_2$	$\mathbb{S}_3$	$\mathbb{S}_4$	**Ranking**
3	5	q-ROFELWA	0.8119	0.3515	0.6404	0.4884	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
		q-ROFELWG	0.8021	0.3357	0.6131	0.4767	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
	50	q-ROFELWA	0.8058	0.3440	0.6364	0.4809	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
		q-ROFELWG	0.8028	0.3419	0.6310	0.4771	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
	500	q-ROFELWA	0.8028	0.3440	0.6313	0.4779	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
		q-ROFELWG	0.8028	0.3440	0.6311	0.4779	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
10	2	q-ROFELWA	0.6280	0.4888	0.5179	0.4999	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
		q-ROFELWG	0.6123	0.4634	0.5035	0.4998	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
20		q-ROFELWA	0.5390	0.4998	0.5014	0.5000	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
		q-ROFELWG	0.5244	0.4921	0.5000	0.5000	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
30		q-ROFELWA	0.5130	0.5000	0.5001	0.5000	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
		q-ROFELWG	0.5052	0.4977	0.5000	0.5000	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$

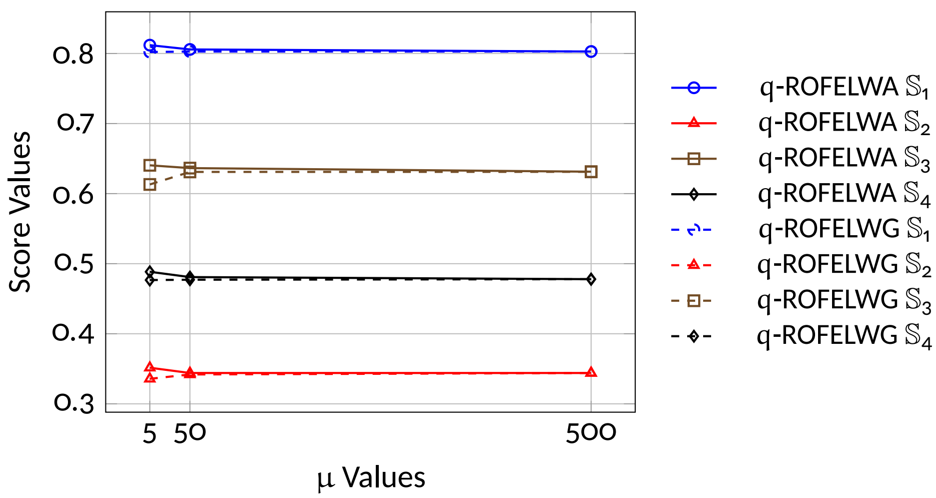

Fig. 2. Score Values and Rankings of Proposed Operators across Fixed q=3 and Different μ Values

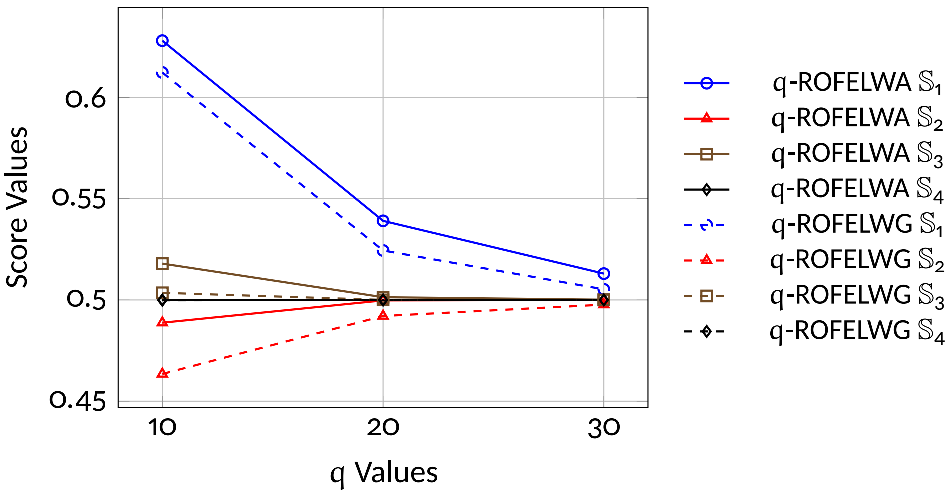

Fig. 3. Score Values and Rankings of Proposed Operators across Different q and Fixed μ=2 Values

3.4 *Comparative evaluation of the proposed AOs*

Table 7 showcases the comparative analysis of score values and rankings for various alternatives using existing and proposed aggregation operators. The results highlight the superiority of our proposed operators, such as q-ROFELWA and q-ROFELWG, which consistently achieve higher score values and provide more accurate and distinct rankings compared to traditional methods like q-ROFFWA, Yq-ROFWA, and q-ROFAAWG. For instance, q-ROFELWA assigns scores of 0.8119, 0.3515, 0.6404, and 0.4885 to $\mathbb{S}_1$, $\mathbb{S}_2$, $\mathbb{S}_3$, and $\mathbb{S}_4$, respectively, outperforming most existing operators. Additionally, our methods excel in handling complex fuzzy information by incorporating enhanced Yager-based operations, allowing flexible parameter tuning and ensuring robustness in distinguishing alternatives. This adaptability enables our operators to balance optimistic and pessimistic criteria effectively, making them more aligned with practical decision-making objectives. Overall, the proposed operators demonstrate their ability to provide a comprehensive evaluation, offering superior precision and applicability in diverse decision-making scenarios. This comparison is illustrated in Figure 4.

Table 7
Comparison with Existing Aggregation Operators

Operator	Score Value $\mathbb{S}_1$	$\mathbb{S}_2$	$\mathbb{S}_3$	$\mathbb{S}_4$	Ranking
q-ROFFWA [6]	0.6050	0.4531	0.5482	0.4964	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFFWG [6]	0.5999	0.4428	0.5342	0.4921	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
Yq-ROFWA [18]	0.1521	0.0003	0.0304	0.0000	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_2 \succ \mathbb{S}_4$
Yq-ROFWG [18]	0.5184	0.4373	0.5055	0.4995	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFEWA[8]	0.8154	0.3600	0.6438	0.4894	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFEWG [8]	0.8003	0.3287	0.6013	0.4766	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFAAWA [48]	0.8397	0.4160	0.7051	0.5021	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFAAWG [48]	0.7444	0.2251	0.5304	0.4578	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFSSWA [51]	0.7811	0.2718	0.6199	0.4611	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFSSWG [51]	0.8180	0.3622	0.6812	0.4798	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFELWA	0.8119	0.3515	0.6404	0.4885	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$
q-ROFELWG	0.8021	0.3357	0.6131	0.4767	$\mathbb{S}_1 \succ \mathbb{S}_3 \succ \mathbb{S}_4 \succ \mathbb{S}_2$

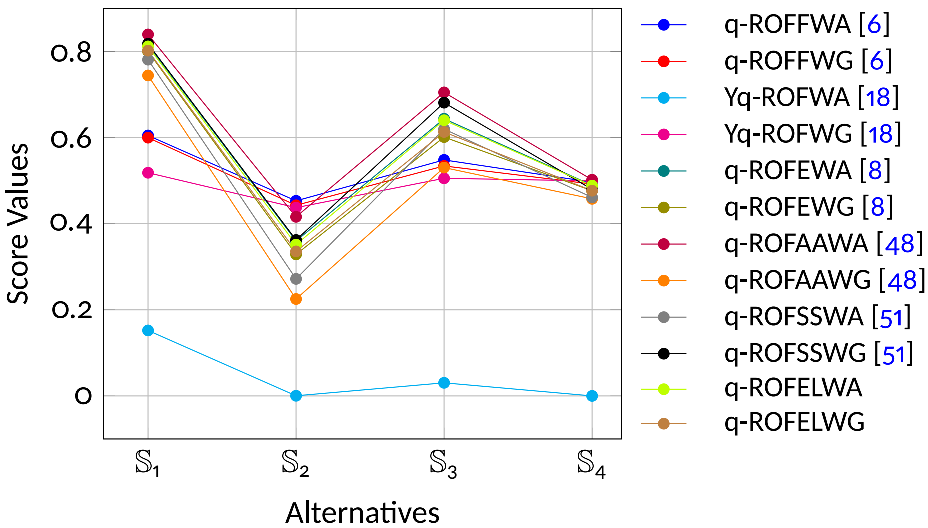

Fig. 4. Comparison with Existing Aggregation Operators

4. Conclusion

In conclusion, our study has significantly advanced the field of q-rung orthopair fuzzy sets by introducing ExpoLogarithmic t-norms and t-conorms, providing powerful tools for handling uncertainty and imprecision in decision-making processes. We have meticulously articulated the fundamental operations of these ExpoLogarithmic aggregation tools, shedding light on their intrinsic mechanisms and offering a robust foundation for their application. The introduction of specialized aggregation operators, namely q-ROFELWA and q-ROFELWG, represents a groundbreaking contribution. These operators demonstrate versatility and efficacy in aggregating q-rung orthopair fuzzy numbers, enriching the decision-making toolkit available to researchers and practitioners. A key highlight of our study lies in the validation of these proposed approaches through a real-life problem on sustainable urban innovation and resilience. Leveraging artificial intelligence in a Multi-Criteria Decision-Making method, we have demonstrated the practical applicability and robustness of our proposed ExpoLogarithmic aggregation operators. Overall, this study not only fills critical research gaps but also presents innovative solutions that can revolutionize decision-making in uncertain and complex environments. The introduced ExpoLogarithmic tools and aggregation operators provide a paradigm shift in handling fuzzy information, offering new avenues for future research and applications in various domains.

4.1 Implications of the proposed study

Our study introduces ExpoLogarithmic t-norms and t-conorms, advancing fuzzy set theory and enhancing uncertainty modeling in various fields. The development of fundamental operations in ExpoLogarithmic aggregation tools deepens understanding, facilitating their practical application in decision-making scenarios. The proposed aggregation operators, q-ROFELWA and q-ROFELWG, provide efficient methods for aggregating q-rung orthopair fuzzy numbers, improving decision processes under uncertainty. Validation through a real-life Multi-Criteria Decision-Making application in sustainable urban innovation highlights the practical significance of our approaches in solving complex challenges.

4.2 Limitations of the proposed study

While impactful, the study has limitations. The generalizability of ExpoLogarithmic t-norms, t-conorms, and aggregation operators across diverse scenarios needs further exploration. The focus on sustainable urban innovation may limit broader applicability. Additional empirical validation and comparative studies are required, particularly to assess performance in large-scale problems. Computational complexity, especially for large-scale decisions, should also be considered. Addressing these limitations will refine the practical application of ExpoLogarithmic tools.

4.3 Future directions of the proposed study

Future research should explore the properties and applications of ExpoLogarithmic t-norms and t-conorms, potentially expanding their capabilities. Optimizing ExpoLogarithmic aggregation operators for large-scale decision-making and validating them across different domains will enhance their effectiveness. Collaboration between researchers and practitioners will bridge theoretical advancements with practical implementation, fostering broader adoption in decision science.

Appendix

Table 8
Abbreviations

Abbreviation	Full Form
AI	Artificial Intelligence
AOs	Aggregation Operators
ELTN	ExpoLogarithmic t-norm
ELTCN	ExpoLogarithmic t-conorm
FS	Fuzzy Set
IFS	Intuitionistic Fuzzy Set
PyFS	Pythagorean Fuzzy Set
MCDM	Multi-Criteria Decision-Making
MV	Membership Value
NMV	Non-Membership Value
q-ROFELAOs	q-rung orthopair fuzzy ExpoLogarithmic aggregation operators
q-ROFELWA	q-rung orthopair fuzzy ExpoLogarithmic weighted average
q-ROFELWG	q-rung orthopair fuzzy ExpoLogarithmic weighted geometric
q-ROFS	q-rung orthopair fuzzy set

Acknowledgement

This research was not funded by any grant.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Peng, Z. R., Lu, K. F., Liu, Y., & Zhai, W. (2023). The pathway of urban planning ai: From planning support to plan-making. *Journal of Planning Education and Research*. https://doi.org/10.1177/0739456X231180568
[2] Allam, Z., & Dhunny, Z. A. (2019). On big data, artificial intelligence and smart cities. *Cities*, *89*, 80–91.
[3] Ali, R., Khattak, A. M., & Iqbal, F. (2023). Multi-criteria decision making (mcdm) using artificial intelligence. *Applied Sciences*. https://www.mdpi.com/journal/applsci/special_issues/I5935856EE
[4] Kamacı, H., Palpandi, B., Petchimuthu, S., & Fathima Banu, M. (2025). M-polar n-soft set and its application in multi-criteria decision-making. *Computational and Applied Mathematics*, *44*(1), 1–46.
[5] Petchimuthu, S., Palpandi, B., & Senapati, T. (2024). Exploring pharmacological therapies through complex q-rung picture fuzzy aczel–alsina prioritized ordered operators in adverse drug reaction analysis. *Engineering Applications of Artificial Intelligence*, *133*, 107996.

[6] Seikh, M. R., & Mandal, U. (2022a). Q-rung orthopair fuzzy frank aggregation operators and their application in multiple attribute decision-making with unknown attribute weights. *Granular Computing*, 1–22. https://doi.org/10.1007/s41066-021-00290-2
[7] Raj, R. G., & Palpandi, B. (2018). Development of fuzzy logic-based speed control of novel multilevel inverter-fed induction motor drive. *Applied Mathematics Information Sciences An International Journal*, *12*, 745–752.
[8] Riaz, M., et al. (2020). A robust q-rung orthopair fuzzy information aggregation using einstein operations with application to sustainable energy planning decision management. *Energies*, *13*(9), 2155.
[9] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1007/978-1-4899-1633-4_2
[10] Atanassov, K. T. (1989). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *33*(1), 37–45.
[11] Jiang, W., & Wei, B. (2018). Intuitionistic fuzzy evidential power aggregation operator and its application in multiple criteria decision-making. *International Journal of Systems Science*, *49*(3), 582–594.
[12] Sirbiladze, G., & Sikharulidze, A. (2018). Extensions of probability intuitionistic fuzzy aggregation operators in fuzzy mcdm/madm. *International Journal of Information Technology Decision Making*, *17*(2), 621–655.
[13] Seikh, M. R., & Mandal, U. (2023). Q-rung orthopair fuzzy archimedean aggregation operators: Application in the site selection for software operating units. *Symmetry*, *15*(9), 1680.
[14] Senapati, T., Simic, V., Saha, A., Dobrodolac, M., Rong, Y., & Tirkolaee, E. B. (2023). Intuitionistic fuzzy power aczel-alsina model for prioritization of sustainable transportation sharing practices. *Engineering Applications of Artificial Intelligence*, *119*, 105716.
[15] Gohain, B., Chutia, R., & Dutta, P. (2022). Distance measure on intuitionistic fuzzy sets and its application in decision-making, pattern recognition, and clustering problems. *International Journal of Intelligent Systems*, *37*(3), 2458–2501.
[16] Ke, Y., Tang, H., Liu, M., & Qi, X. (2022). A hybrid decision-making framework for photovoltaic poverty alleviation project site selection under intuitionistic fuzzy environment. *Energy Reports*, *8*, 8844–8856.
[17] Wan, S. P., & Yi, Z. H. (2016). Power average of trapezoidal intuitionistic fuzzy numbers using strict t-norms and t-conorms. *IEEE Transactions on Fuzzy Systems*, *24*, 1035–1047.
[18] Yager, R. R. (2013). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, *22*(4), 958–965.
[19] Akram, M., Dudek, W. A., & Dar, J. M. (2019). Pythagorean dombi fuzzy aggregation operators with application in multicriteria decision-making. *International Journal of Intelligent Systems*, *34*(11), 3000–3019.
[20] Shahzadi, G., Akram, M., & Al-Kenani, A. N. (2020). Decision-making approach under pythagorean fuzzy yager weighted operators. *Mathematics*, *8*(1), 70.
[21] Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *25*(5), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005
[22] Wang, J., Wei, G., Wei, C., & Wei, Y. (2020). Mabac method for multiple attribute group decision making under q-rung orthopair fuzzy environment. *Defence Technology*, *16*(1), 208–216.
[23] Seikh, M. R., & Mandal, U. (2022b). Q-rung orthopair fuzzy frank aggregation operators and its application in multiple attribute decision-making with unknown attribute weights. *Granular Computing*, 1–22.
[24] Wang, J., Zhang, R., Zhu, X., Zhou, Z., Shang, X., & Li, W. (2019). Some q-rung orthopair fuzzy muirhead means with their application to multi-attribute group decision making. *Journal of Intelligent and Fuzzy Systems*, *36*(2), 1599–1614.

[25] Wang, J., Wei, G., Lu, J., Alsaadi, F. E., Hayat, T., Wei, C., & Zhang, Y. (2019). Some q-rung orthopair fuzzy hamy mean operators in multiple attribute decision-making and their application to enterprise resource planning systems selection. *International Journal of Intelligent Systems*, *34*(10), 2429–2458.

[26] Kausar, R., Farid, H. M. A., Riaz, M., & Gonul Bilgin, N. (2023). Innovative codas algorithm for q-rung orthopair fuzzy information and cancer risk assessment. *Symmetry*, *15*, 205. https://doi.org/10.3390/sym15010205

[27] Kabir, S., & Papadopoulos, Y. (2018). A review of applications of fuzzy sets to safety and reliability engineering. *International Journal of Approximate Reasoning*, *100*, 29–55.

[28] Mardani, A., Hooker, R. E., Ozkul, S., Yifan, S., Nilashi, M., Sabzi, H. Z., & Fei, G. C. (2019). Application of decision making and fuzzy sets theory to evaluate the healthcare and medical problems: A review of three decades of research with recent developments. *Expert Systems with Applications*, *137*, 202–231.

[29] Pinar, A., & Boran, F. E. (2022). A novel distance measure on q-rung picture fuzzy sets and its application to decision making and classification problems. *Artificial Intelligence Review*, *55*(2), 1317–1350.

[30] Ngan, R. T., Ali, M., Fujita, H., Abdel-Basset, M., Giang, N. L., Manogaran, G., & Priyan, M. K. (2019). A new representation of intuitionistic fuzzy systems and their applications in critical decision making. *IEEE Intelligent Systems*, *35*(1), 6–17.

[31] Haq, I. U., Shaheen, T., Ali, W., & Senapati, T. (2022). A novel sir approach to closeness coefficient-based magdm problems using pythagorean fuzzy aczel-alsina aggregation operators for investment policy. *Discrete Dynamics in Nature and Society*, Article ID 5172679, 12 pages. 10.1155/2022/5172679

[32] Dey, A., Senapati, T., Pal, M., & Chen, G. (2022). Pythagorean fuzzy soft rms approach to decision making and medical diagnosis. *Afrika Matematika*, *33*, 97. 10.1007/s13370-022-01031-7

[33] Rani, P., Mishra, A. R., Krishankumar, R., Ravichandran, K. S., & Gandomi, A. H. (2020). A new pythagorean fuzzy based decision framework for assessing healthcare waste treatment. *IEEE Transactions on Engineering Management*, *69*(6), 2915–2929.

[34] Kamacı, H., & Petchimuthu, S. (2022a). Soergel distance measures for q-rung orthopair fuzzy sets and their applications. In *Q-rung orthopair fuzzy sets: Theory and applications* (pp. 67–107). Springer.

[35] Kamacı, H., & Petchimuthu, S. (2022b). Some similarity measures for interval-valued bipolar q-rung orthopair fuzzy sets and their application to supplier evaluation and selection in supply chain management. *Environment, Development and Sustainability*, 1–40.

[36] Senapati, T., Martínez, L., & Chen, G. (2023). Selection of appropriate global partner for companies by using q-rung orthopair fuzzy aczel-alsina average aggregation operators. *International Journal of Fuzzy Systems*, *25*, 980–996.

[37] Fathima Banu, M., Petchimuthu, S., Kamacı, H., & Senapati, T. (2024). Evaluation of artificial intelligence-based solid waste segregation technologies through multi-criteria decision-making and complex q-rung picture fuzzy frank aggregation operators. *Engineering Applications of Artificial Intelligence*, *133*, 108154. https://doi.org/10.1016/j.engappai.2024.108154

[38] Petchimuthu, S., Banu M, F., Mahendiran, C., & Premala, T. (2025). Power and energy transformation: Multi-criteria decision-making utilizing complex q-rung picture fuzzy generalized power prioritized yager operators. *Spectrum of Operational Research*, *2*(1), 219–258. https://doi.org/10.31181/sor21202525

[39] Liu, P., & Wang, P. (2018). Multiple-attribute decision-making based on archimedean bonferroni operators of q-rung orthopair fuzzy numbers. *IEEE Transactions on Fuzzy Systems*, *27*(5), 834–848.

[40] Wei, G., Gao, H., & Wei, Y. (2018). Some q-rung orthopair fuzzy heronian mean operators in multiple attribute decision making. *International Journal of Intelligent Systems*, *33*(7), 1426–1458.

[41] Garg, H., & Chen, S. M. (2020). Multiattribute group decision making based on neutrality aggregation operators of q-rung orthopair fuzzy sets. *Information Sciences*, *517*, 427–447.

[42] Xing, Y., Zhang, R., Wang, J., Bai, K., & Xue, J. (2020). A new multi-criteria group decision-making approach based on q-rung orthopair fuzzy interaction hamy mean operators. *Neural Computing and Applications*, *32*, 7465–7488.

[43] Deveci, M., Pamucar, D., Cali, U., Kantar, E., Kölle, K., & Tande, J. O. (2022). Hybrid q-rung orthopair fuzzy sets based cocoso model for floating offshore wind farm site selection in norway. *CSEE Journal of Power and Energy Systems*, *8*(5), 1261–1280.

[44] Rawat, S. S., & Komal. (2022). Multiple attribute decision making based on q-rung orthopair fuzzy hamacher muirhead mean operators. *Soft Computing*, *26*(5), 2465–2487.

[45] Farid, H. M. A., & Riaz, M. (2023). Q-rung orthopair fuzzy aczel–alsina aggregation operators with multi-criteria decision-making. *Engineering Applications of Artificial Intelligence*, *122*, 106105.

[46] Jabeen, K., Khan, Q., Ullah, K., Senapati, T., & Moslem, S. (2023). An approach to madm based on aczel-alsina power bonferroni aggregation operators for q-rung orthopair fuzzy sets. *IEEE Access*.

[47] Qiyas, M., Abdullah, S., Khan, N., Naeem, M., Khan, F., & Liu, Y. (2023). Case study for hospital-based post-acute care-cerebrovascular disease using sine hyperbolic q-rung orthopair fuzzy dombi aggregation operators. *Expert Systems with Applications*, *215*, 119224.

[48] Khan, M. R., Wang, H., Ullah, K., & Karamti, H. (2022). Construction material selection by using multi-attribute decision making based on q-rung orthopair fuzzy aczel–alsina aggregation operators. *Applied Sciences*, *12*(17), 8537.

[49] Jacintos Nieves, A., & Delgado Ramos, G. C. (2023). Advancing the application of a multidimensional sustainable urban waste management model in a circular economy in mexico city. *Sustainability*, *15*(17), 12678.

[50] Liu, P., & Liu, J. (2018). Some q-rung orthopair fuzzy bonferroni mean operators and their application to multi-attribute group decision making. *International Journal of Intelligent Systems*, *33*(2), 315–347.

[51] Rong, Y., Li, Q., & Pei, Z. (2020). A novel q-rung orthopair fuzzy multi-attribute group decision-making approach based on schweizer-sklar operations and improved copras method. *Proceedings of the 4th International Conference on Computer Science and Application Engineering*, 1–6.